CAMTE Monograph 1

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Secondary Mathematics Methods Courses in California

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Preface

In one sense, this monograph marks the end of a long road, but it is more important that it be seen as a beginning. The trip that led to the creation of this document began when I asked an innocent question, “What should be the content of a secondary mathematics methods course in California?” I was relatively new to California at the time and was asked to teach such a course as part of my assignment at CSU Bakersfield. It seemed to me as if our new organization, the California Association of Mathematics Teacher Educators (CAMTE) would/should be an obvious source of that information. The response to my question was, in essence, “That is a great question! Why don’t you form a task force to determine the answer?” The response to our initiative was great. We began with well-attended working group sessions at the CMC Asilomar Conference and a PMET-sponsored conference that Dale Oliver was able to arrange. The group decided that it would be a good idea to create a monograph that would provide a research-based description of what such a course should contain. The first step was to learn what was currently being done in such courses across the state. As you can read in Chapter One by Margaret Kidd, we learned that there is little uniformity in secondary mathematics methods courses across California, which is consistent with what happens in those courses across the country (Taylor & Ronau, 2006).

Accordingly, this monograph does not answer the question that it was initially intended to answer. It does not describe all of the content that should be included in methods courses. For example, there is no chapter on technology, a critical topic for preservice secondary mathematics teachers. What the authors of the six chapters are giving us are important perspectives on topics that would be expected to be included in methods courses. We hope that our monograph will serve as a valuable resource for anyone teaching or about to teach a methods course and as a catalyst for conducting more research and sharing of expertise. The authors and I are including our email addresses and encourage you to contact us with questions, with suggestions, and with examples of how you use the content of the monograph.

As would be expected on such a large project, there are many people who should be thanked. Heather Calahan and Rick Marks helped plan the CAMTE Secondary Mathematics Methods Task Force Conference at Asilomar in June, 2006, worked hard to put out our call for monograph proposals, and helped review submissions. Dale Oliver's grant provided funds to support the conference in Asilomar. Brian Lim created and has maintained a website for our project at California State University, Sacramento (http://edweb.csus.edu/projects/camte/). There were 27 people who attended the Asilomar conference and many more who attended the task force sessions at the CMC conferences in Asilomar. Thanks to Viji Sundar, Mary Jo Anhalt, Debasree Raychaudhuri, and Joanne Becker for their work as reviewers. All of them as well as the CAMTE officers who supported the project deserve thanks with special appreciation to my colleague, Terran Felter, for her diligent editing assistance. And of course, thanks to the authors of our six chapters!!!

We hope you find our work beneficial.

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1

A Comparison of Secondary Mathematics Methods Courses in California

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In 2004 California Governor Arnold Schwarzenegger made a "compact" with California State University (CSU) and the University of California (UC), which called on them to work together to develop a program to prepare more teachers in mathematics and science. The governor pledged $1.25 million for this program in his 2005-06 state budget and over $2.8 million in his 2006-07 budget (the 2008-2009 budget is not finalized at this time). As a result of this agreement, on May 31, 2005, California Governor Arnold Schwarzenegger, California State University Chancellor Charles B. Reed and University of California President Robert C. Dynes announced the creation of a new program to prepare more math and science teachers for the state's K-12 schools.

The University of California instituted the California Teach Science-Mathematics Initiative (CaTEACH-SMI). Under this program, the UC system will quadruple its annual production of credentialed science and mathematics teachers, from 250 per year to 1,000 per year by 2010. This initiative is the largest of its kind in the nation.

The CSU system joined in the effort by expanding its own teacher preparation programs for science and mathematics teachers as well as its recruitment of students to the profession. California State University’s goal is to at least double the production of math and science teachers during the next five years. The CSU is the state's largest producer of teachers, and, through this new program, will expand its existing capacity to recruit and prepare teachers in these critical shortage areas. This means increasing from a baseline figure of approximately 750 new math and science teachers produced annually to a figure of 1,500 new teachers produced in these fields.

During the December 2005 California Mathematics Council (CMC) Conference, interest was sparked by a session on “What Should Be Taught in a Methods Course?” From the conversation during the question and answer period, it was decided that further conversation was warranted. The following spring, organizers developed a conference, June 6, 2006 on the Asilomar Campgrounds. Due to the interest garnered during the conference, a network of the Math

Methods instructors was created. The next step was to gather and post each of the methods syllabi so that they could be shared by all.

This study is a review of the syllabi of these courses at the CSUs and UCs. With the intense emphasis on increasing the number of math teachers, it seems natural to ask if the graduates of these programs are receiving relatively comparable educations. Are the courses similar or emphasize the same elements? What *is* being taught in these courses? Is there a basis for conversations and sharing of ideas?

The Study

This paper concentrates on comparing elements the courses have in common. The hope is that all of the courses will be better by incorporating what others see as important. This study is based, in part, on the work of Taylor and Ronau (2006). Although they conducted a nationwide survey of all grade bands, this analysis concentrates only on California Secondary Mathematics. They tried to answer the following questions:

1. What are the common elements of mathematics methods courses?
2. What elements might encourage the development of leadership skills?
3. What elements might lead to increased capacity and/or inclination to collaborate?
4. What elements contribute to a commitment to continual professional development?
5. How might these elements vary among methods courses for different grade levels?

Due to the limited scope of this study only the first question was investigated, the emphasis being on Secondary Education and on what is common amongst the programs. Of the 23 CSU and 9 UC campuses, 14 of the former and 2 of the latter's syllabi were submitted.

Commonalities

Surprisingly, there were very few elements in common. After analyzing the syllabi the following areas were found mentioned most often: 1) written Lesson Plans and unit plans; 2) reading or doing research; 3) reading and reflecting on outside articles; 4) textbook use; 5) NCTM membership requirement; 6) College in which the course is housed (Education or Math); 7) learning or using technology; 8) learning about or using manipulatives; 9) formal portfolio or written Philosophy of Mathematics Education; 10) class presentations of mini lessons.

The only element all of the courses had in common was attendance/participation. Most noted that attendance was mandatory in their syllabus. Many further noted that students would not be able to complete the course if more than one absence occurred. Others included marking the final grade one letter lower for more than one absence. Many areas, such as where a course was housed, use of research and formal assessment, and learning about technology were evenly split.

The second most similar activity, submission of written Lesson Plans, was required in all but two courses. In addition to Lesson Plans, eight further required writing unit or other long range plans. The majority, eleven, required writing reflections on outside readings. Seven of the syllabi contained references to reading research articles. Twelve used outside readings of some type, some in addition to the research articles. Only two of the sites used a textbook exclusively. One of these sites used the Huetinck text (2004) and the second used the Brahier (2008).

To summarize some of the other commonalities, of the sixteen syllabi, nine required membership in the National Council of Teachers of Mathematics (NCTM). Nine also used NCTM's 2000 *School Mathematics Principles and Standards for School Mathematics* (*PSSM*) as part of the required or recommended reading list. Not surprisingly, twelve also mentioned the use of the *California Standards*. The two textbooks that were used most frequently were the Huetinck, used by four instructors, and Brahier, required for two courses. Four other sites used different texts and two printed their own course packets. The final site did not mention use of a text or course packet. Four syllabi included web site addresses as additional information for the students.

Seven of the programs were housed in the Department of Mathematics while eight were housed in the College of Education and one had dual housing in both colleges. Nine include either Field work or observation in conjunction with the course. One conclusion that might be inferred from this is that there is little consistency as to the placement of the methods course on the timeline of the credential program. This is an area of further investigation and discussion.

Technology is not mentioned in eight of the syllabi. The other half had students either write a lesson plan incorporating technology or had designated classes in technology. The specific technological references were to calculators and Geometer's Sketchpad. Some courses seemed to take the use of technology for granted as they referred to it in the generic term. Use of manipulatives was not mentioned in eleven of the courses. Again, in the six courses that did use manipulatives, students frequently had to write lesson plans specifically targeting them.

Assessment

A formal portfolio of Philosophy of Mathematics Education was required as a culmination of the experience in only six of the courses. The other ten did require some type of journal writing, packet of activities or final compilation of reaction papers.

All courses included attendance and participation in their final grade. One instructor conducted an individual interview as the final assessment. Eight of the others had a formal exam given during the normal final week but seven did not have any type of final exam. The *Teacher Performance Expectations* were explicitly referred to in eight syllabi and one still used the 1997 document, *California Standards for the Teaching Profession*. This raises the question as to whether all programs require the Teacher Performance Assessment or some other form of standardized state assessment. Nine of the syllabi spelled out student outcomes or student dispositions in detail. The others did not mention them.

All other elements that were observed were found sporadically throughout the courses. They were not used by more than one or two programs so have been ignored for this study.

Conclusions

Few conclusions can be definitively drawn from this detailed study of the syllabi. However, it is a starting point for conversations that should enrich all of the courses and should result in a more broadly educated future teacher.

In general, all of the courses prepare students in understanding how to teach students in a public high school. The "methods" employed to do this, however, varied widely. The courses all do generally follow the California Commission on Teacher Credentialing guidelines, or TPEs, which describe teaching tasks that fall into six broad domains: 1) creating and maintaining effective environments for student learning; 2) making subject matter comprehensible to students; 3) assessing student learning; 4) engaging and supporting students in learning; 5) planning instruction and designing learning experiences for students; and, 6) developing as a professional educator.

With the exception of two, all courses required writing lesson plans and some even more detailed, long term plans. Since this is so basic to the duties of a teacher, it should not be surprising that educators want to train the beginning teachers in the basic elements that are necessary to perform their job. The majority of the programs requires or suggests membership in the National Council of Teachers of Mathematics as well as the study of its guidelines, *PSSM*.

This result should not be surprising in that the NCTM sets the national standards for mathematics instruction. This provides students a broader concept of what is being done in states other than California. Others require, recommend or provide membership in the California Mathematics Council and attendance at local or regional math conferences. Membership in the state organization and attendance at conferences provides these students with a deeper understanding of what is occurring in other areas of the state. Naturally, a detailed study of the specific grade levels in *The California Framework for California Public Schools: Kindergarten through Twelve* is included in most of the courses. There is accountability for each of the courses although some are offered for a letter grade and others simply for Credit.

Some of the syllabi give a more detailed outline of their course than others, so a more complete picture of these could be seen than from those who were more general. A more comprehensive sense would be garnered if all sites participated. It was unfortunate that only sixteen of the thirty-two programs responded to the request. Many avenues, including both a verbal and written request to all of the CSU Mathematics Department Chairs, were employed for more than a year to solicit responses. In order to fully understand what is being taught in methods courses, resources other than syllabi need to be considered. In spite of the limitations of this study, it is hoped that it will motivate deeper discussions on the role Methods courses play in the education of student teachers. It is further hoped that all sites will be encouraged to participate in this conversation.

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Curricular Knowledge for Secondary Mathematics Teachers

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Abstract

How curricular knowledge (Shulman, 1986) for preservice secondary mathematics teachers develops receives very little attention in the literature regarding the preparation of mathematics teachers, yet many activities within mathematics teacher education courses implicitly, if not explicitly, attend to curricular knowledge. The purpose of this paper is to expand upon Shulman's (1986) notion of curricular knowledge in the context of mathematics teacher preparation. This discussion draws from 1) specific activities and assignments from secondary methods courses intended to initiate the development of curricular knowledge, 2) research about teacher thinking, and 3) frameworks for characterizing teachers' uses and interactions with reform curricula.

Teachers' uses and interactions with mathematics reform curriculum materials is an emerging area of study (Ball & Feiman-Nemser, 1988; Lloyd, 1999, 2008; Remillard, 1999; Remillard & Bryans, 2004; Sherin & Drake, in press), as is how reform curricula can be used as catalysts for mathematics teacher learning in both the preservice and in-service arenas (Frykholm, 2005; Lloyd, 2006; Remillard, 2000; Tarr & Papick, 2004). In part, this work speaks to Shulman's (1986) notion of curricular knowledge for teachers, particularly as teachers and student-teachers decide what materials to use and how to use them. Consider, for example, Alyssa's description of how she planned a unit in geometry for her third graders:

> *At first I just sat down and off the top of my head thought about, "What do I want them to learn when it comes to geometry?" – and I just wrote down some ideas. Then I start piling all sorts of resources together. I read through the section on geometry in the Standards; I read through a lot of curriculum guides on geometry, and through textbooks, to see what they cover. I look at different resource materials, for example, from Marilyn Burns. And I think*

of ideas of activities. So I have a broad plan throughout the unit of the concepts I want covered, possible activities, and then I sort of plot them in. I throw all my files and all my resource materials on geometry together in a box, and I keep that and look through it for lessons and ideas and take out what I need. (Philipp, Flores, Sowder & Schappelle, 1994, p. 173)

Clearly Alyssa, an experienced teacher, has a decision-making process for sorting through her materials. Preservice teachers, however, lack the knowledge and experience needed to make such decisions. Accordingly, mathematics teacher preparation courses engage preservice teachers in activities such as lesson and textbook analysis, intended to teach about curricular knowledge (Ball & Feiman-Nemster, 1988; Lloyd & Behm, 2005**)**. But little is known about how this knowledge develops, particularly at the preservice level.

The purpose of this paper is to expand upon Shulman's (1986) notion of curricular knowledge in the context of mathematics teacher preparation. This discussion draws from 1) specific activities and assignments from secondary methods courses intended to initiate the development of curricular knowledge, 2) research about teacher thinking, and 3) frameworks for characterizing teachers' uses and interactions with reform curricula to address the following questions:

- What is curricular knowledge for mathematics teachers?
- In what ways can instructors of mathematics teacher education courses foster development of curricular knowledge for mathematics teachers?
- In what ways does curricular knowledge develop for preservice mathematics teachers?

Curricular knowledge, as described by Shulman (1986) is knowledge of the full range of programs designed for the particular teaching of subjects and topics at a given level, the variety of instructional materials available in relation to those programs, and the set of characteristics that serve as both the indications and contraindications for the use of particular curriculum or program materials in particular circumstances

. . . [We] would also expect a professional teacher to be familiar with [lateral curricular knowledge,] the curriculum materials under study by his or her students in other subjects they are studying at the same time. … The vertical equivalent of that curriculum knowledge is familiarity with the topics and issues that have been and will be taught in the same subject

area during the preceding and later years in school, and the materials that embody them. (p. 10).

Accordingly, curricular knowledge for K-12 mathematics teachers can be considered as comprised of four components: 1) knowledge of different curricula and corresponding materials available for teaching mathematics at a given grade level, 2) knowledge of how to compare, contrast and modify curricula for given contexts, including knowledge of research that supports and does not support the use of such curricula, 3) knowledge of content in other subject areas of students (lateral curricular knowledge), and 4) knowledge of how mathematics topics are developed across the grade levels (vertical curricular knowledge).

Alyssa's comments cited earlier indicated an awareness of the materials available to her (component 1), and she has a process for deciding which materials go in the box (component 2), although the specifics of this process are not shared in her description. Components 2 and 4 are illustrated more explicitly in the decision-making processes of Mrs. Hanson and her colleagues, a group of sixth grade teachers (Boston, Smith, & Hillen, 2003) who used research about children's development of proportional reasoning (Langrall & Swafford, 2000) to plan ways for students to develop the cross-product method from informal strategies. They strove to "weave ratio and proportion problems throughout the sixth-grade curriculum rather than isolate these ideas in a single unit at the end of the year, which was the approach taken by the textbook" (Boston et al, 2003, p. 151). Here, we see evidence of teachers modifying resources (the textbook) based on research about children's thinking (component 2), with a focus on how to develop proportional reasoning concepts over the course of a year (component 4).

These examples involve experienced practicing mathematics teachers. Returning to the preservice arena, Table 1 offers five activities within mathematics teacher preparation courses that can facilitate the development of curricular knowledge for preservice teachers.

Although states typically provide truncated versions of the same idea, it is useful to have preservice teachers create their own K-12 posters of how specific concepts are developed across the state-mandated objectives followed by an analysis of the poster in light of mathematics education research. For example, using the state-mandated objectives, preservice teachers can create a K-12 poster for geometry and measurement concepts and then analyze the poster in terms of the van Hiele levels (Fuys, Geddes, & Tischler, 1988) and national standards for teaching geometry and measurement (National Council of Teachers of Mathematics, 2000).

Such an activity addresses components 1, 2, and 4 of curricular knowledge. First, the activity initiates awareness and examination of the state-mandated objectives, where to find them, and how to interpret them. Second, the activity provides guidance in developing criteria, such as research about children's thinking, one might use to evaluate curricula (in this case, the state-mandated objectives). Third, in Shulman's (1986) words, this activity instills "familiarity with the topics and issues that have been and will be taught in the same subject area during the preceding and later years in school" (p. 10), as expected by the state and compared to national recommendations.

Activity	Goals	Curricular Knowledge Components
1) K – 12 poster of a given set of concepts, as represented in the state-mandated objectives *2) Analysis of state-mandated assessments*	• Initiate examination of state-mandated objectives • Instill a critical mindset in reviewing materials (evidence based in mathematics education research) • Development of vertical curricular knowledge	1, 2, 4
3) Locating, analyzing, and sharing of self-chosen professional articles *4) Professional Resource Binder*	• Increased awareness of available ideas and materials • Development of skills in reading and analyzing resources • Development of lateral curricular knowledge	1, 2, 3
5) Lesson critiquing/modifying	• Instill a critical mindset in reviewing materials (evidence based in mathematics education research)	2

Table 1

A second, similar activity is to provide different groups of students with different grade level state-mandated released assessments. The task is for preservice teachers to identify items that assess one of the state objectives, say algebraic reasoning. Along with meaningful discussions about assessment (Chauvot & Benson, 2008), fruitful discussions emerge in 'tracking" what algebraic reasoning is expected to look like at different grade levels, and whether or not these expectations are consistent with what is recommended in the literature.

A third activity is for students to locate and then analyze professional articles. The beginning of this activity occurs early in the semester where students submit five to ten professional articles on the password-protected electronic discussion group. Component 1 is addressed here in that preservice teachers become increasingly aware of the ideas and materials that are available to them. As they browse the internet, the professional journals, and the articles posted by their peers, preservice teachers begin to appreciate that this is only "the tip of the iceberg."

Next, on a weekly basis, preservice teachers choose a posted article and submit a written analysis of the article. The writing assignment is critical because it alerts the mathematics teacher educator to how preservice teachers are reading and interpreting the articles (e.g., Lloyd & Behm, 2005; Remillard 1999; Sherin & Drake, in press). For example, Sherin & Drake (in press) noted three general approaches used by teachers in reading curricular materials: reading for a broad overview, reading for details, and reading for both. Alyssa (Philipp et al, 1994) generated a "broad plan" and then "sort of plotted them in", whereas Jan, Kate, and Shelley (Sherin & Drake, in press) looked for specific details. This finding is significant for working with preservice teachers as teacher educators design appropriate writing prompts for their students. Furthermore, the writing assignment provides opportunity for one-on-one dialogue between the teacher educator and student. For example, one of my students, in reference to Bright, Joyner, & Wallis (2003) commented:

> *Okay, they did put the following into the paper "it is important to create settings in which they (the students) can apply additive and multiplicative reasoning both correctly and incorrectly." (pp. 166-67). I guess my question or concern about this statement is that I do not think that a testing situation where a student's grade is on the line is the appropriate place for such non-specific questions. Wouldn't it be more prudent to have such questions built into an exploratory activity or during the formulation of the construct instead? Let me know what you think of this please.*

My response was to expand upon what had been addressed in class:

> *I think instruction must align with assessment. Within my instruction, I provide "non-specific" questions to help develop a concept (what it is and what it is not), to help develop problem solving skills, and to help develop flexible thinking. It follows that "non-specific" questions should arise in testing situation as well to assess the objectives I just listed. If it occurs within instruction on a regular basis, students will not be threatened in grading*

situations. If a teacher provides "non-specific" questions within exploratory activities within instruction and does NOT provide the same in testing situations, students infer that exploring and problem-solving is not important and not valuable enough to merit credit on a test.

Finally, although not initially intended, this activity also leads to development of lateral curriculum knowledge (component 3); students always find multiple articles that help them see how the content they will be teaching relates to other subject areas.

A fourth related assignment that attends to the same three components of curricular knowledge is the creation of a “Professional Resource Binder.” This collection of materials, much like Alyssa’s box (Philipp et al, 1994), can be in the form of a three-ring binder, or it can be developed electronically. The intention of the assignment is for students to systematically organize their materials, including the articles, in a way that is meaningful to them that can then be used as practicing teachers.

A fifth activity is within the context of lesson planning, and it focuses on “the set of characteristics that serve as both the indications and contraindications for the use of particular curriculum or program materials in particular circumstances” (Shulman, 1986, p. 10). After addressing components of a lesson plan, small groups of students receive 3-5 lesson plans, the feedback form in figure 1, and instructions to collaboratively critique the lesson plans. The focus of the whole-group discussions about this assignment is the rationales for the decisions that are made in evaluating the lessons. Students are pushed to explain their thinking and provide examples from the lesson to illustrate their decisions. The goal is to initiate the development and understanding of criteria mathematics teachers need to assess curricular materials.

The activities provided highlight ways in which mathematics teacher educators can initiate the development of components of curricular knowledge for preservice teachers. Arguably, components 1, 3 & 4 are not as difficult to address in mathematics teacher education coursework as component 2. Component 2 creates a significant challenge for mathematics teacher educators, particularly in light of research about mathematics teacher beliefs and orientations toward authority (e.g., Cooney, Shealy, & Arvold, 1998; Mewborn, 1999; Philipp, 2007), teacher interactions with mathematics reform curricula (e.g., Lloyd, 1999; Remillard & Bryans, 2004, Sherin & Drake, in press), and discussions about conceptions of curriculum use (Remillard, 2005).

For example, consider Mewborn's (1999) study with four preservice elementary teachers. Within her investigation about teachers' reflective thinking in a field-based mathematics methods course, patterns regarding who had the authority to generate, reason about, and test hypotheses about mathematics teaching and mathematics learning emerged. She identified three distinct stages that characterized shifts in the locus of authority. The first stage was characterized as external where Carrie, Hanna, Emily, and Ashleigh tended to only state problems and look toward an external authority for generating and reasoning about hypothetical solutions to problems. They accepted the words of the perceived authorities (the cooperating teacher and researcher). In the second stage, the locus of authority became both internal and external. The four preservice teachers felt empowered to not only state the problems but also to generate hypotheses for solving problems. They turned toward external authority for help in reasoning about the hypotheses, and used their prior experiences and beliefs as explanations for their ideas. Finally, in the third stage, the preservice teachers relied on themselves and one another to search for evidence within children's mathematical thinking to reason about their hypotheses.

Seen through a lens of developing curricular knowledge for preservice teachers, with a supportive, collaborative environment similar to the environment provided by Mewborn (1999), it is feasible that preservice mathematics teachers may proceed through similar phases as they learn how to critically assess curricula materials, one in which they initially turn toward external authorities for understanding and applying criteria for evaluating materials until they eventually learn to rely on themselves, their colleagues, children's mathematical thinking, and meaningful mathematics.

Cooney et al's (1998) work with four preservice secondary mathematics teachers revealed how each participant's orientation toward authority played a part in their ability to engage in reflective activity. For example, Greg looked to others as well as himself for sources of evidence of what it might mean to be a good mathematics teacher. Greg's propensity to integrate his views and the views of others identified him as reflective connectionist.

Henry, on the other hand, rejected ideas of reform based on personal experience. Although Henry looked to others for approval, he only looked toward those whose ideas were consistent with his. Henry's tendency to hold new ideas "at bay", seeking confirmation for only his own views, and his tendency to hold his beliefs in isolation of one another characterized him as an isolationist.

Nancy strove toward pleasing others. She did what was expected of her. She valued the views of others but assumed that all of their knowledge was the same. Reflective thought centered on looking for harmony among different interpretations of the same knowledge. Nancy's uncritical acceptance of ideas of reform and her tendency to turn to others for knowledge characterized her as a naïve idealist.

Sally at times expected to be told how to teach but struggled between recognizing her own voice and hearing the voices of others. Her reflective activity centered on resolving tensions and integrating voices. Realizing the complexity of teaching, and unable to resolve some of her tensions, Sally left the profession. Sally's tendency to look outward for sources of evidence of knowledge characterized her as a naïve connectionist.

As mathematics teacher educators work toward assisting preservice teachers in learning how to critically analyze materials, they need to be cognizant of various orientations toward authority that preservice teachers may hold. Nancy's responses on the form in Figure 1 may look very similar to Greg's responses. However, Nancy's tendencies toward pleasing others might result in "learning the language of reform" (Wilson & Goldenberg, 1998) whereas Greg's response may be more representative of what he is actually thinking.

Finally, Remillard's (2005) analysis of assumptions and theoretical perspectives that influence conceptions of curriculum use, although focused on researchers' perspectives, is worth considering in the realm of mathematics teacher preparation. A mathematics teacher educator's socio-cultural view of the teacher-curriculum relationship as participatory, influenced by both teacher and curriculum, will generate instructional activities noticeably different from a positivist view that curriculum can be enacted with full fidelity. It is likely that conceptions of curriculum use will differ in any given mathematics teacher preparation course. Mathematics teacher educators need to be mindful of the epistemological views held by the their students and provide a balance of challenge and support to connect students from their current epistemological views to consider more complex ways of thinking (Baxter Magolda, 1992, King & Kitchener, 1994) that is required to be critical consumers of mathematics curriculum.

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Lesson Plan Feedback Form

Lesson by: ______________________________

Evaluation by: ______________________________

Components:

I – incomplete (Not provided or I have questions),
S – satisfactory (Met requirements)

LESSON OVERVIEW	
	Learning objectives and goals clearly stated
	Relevant TEKS listed
	Materials needed
INSTRUCTOR'S NOTES	
	Introductory activity is provided
	Intended lesson structure (individual work; small-group work; whole-class discussion …) is provided
	Instructional strategies for individual accountability are provided
	Instructional strategies for small-group accountability, if applicable, are provided
	Intended examples and solutions are provided
	Appropriate assessment strategies are indicated
	A corresponding homework assignment, if applicable, is provided
	Wrap-up is provided
	Items pertinent to the lesson that might be uses on a unit test are provided
	Relevant TAKS items are provided

Criteria:

1. Examine the introductory activity. Place an X on the line below that best describes your thoughts regarding the activity.

Intro activity was effective in initiating students into the lesson	______________________	Intro activity was not effective in initiating students into the lesson

Rationale:

____

____

2. Place an X on the line below that best describes your thinking in terms of student engagement of the lesson provided.

Instructional strategies support student engagement	________________________	Instructional strategies do not support student engagement

Rationale:

____

____

3. Place an X on the line below that best describes your thinking in terms of the mathematics of the main activities of the lesson

The mathematics is not "meaningful".	________________________	The mathematics is "meaningful".

Rationale:

____

____

4. Place an X on the line below that best describes your thinking in terms of opportunities for students to communicate mathematical thinking to peers and the teacher.

Opportunities for students to communicate were evident in the lesson	________________________	Opportunities for students to communicate were not evident in the lesson

Rationale:

____

____

5. Place an X on the line below that best describes your thinking in terms of opportunities for students to make connections between different representations concepts.

Opportunities for students to make connections were evident in the lesson	________________________	Opportunities for students to make connections were not evident in the lesson

Rationale:

____

____

6. Place an X on the line below that best describes your thinking in terms of the lesson "wrap-up"

Instructional strategies were effective in "wrapping up" the lesson	______________________	Instructional strategies were not effective in "wrapping up" the lesson.

Rationale:

____
____

7. Place an X on the line below that best describes your thinking in terms of the homework assignment, if applicable.

The hw assignment was appropriate for this lesson.	______________________	The hw assignment was not appropriate for this lesson

Rationale:

____
____

8. Place an X on the line below to indicate your rating of the unit exam items.

Mostly conceptual. Students must understand material to do well.	______________________	Mostly procedural or rule-based. Students need to know rules to do well.

Rationale:

____
____

9. This lesson was/was not attentive to what we know about how students learn mathematics
____
____
____

10. This lesson was/was not attentive to students making sense of procedures because
____
____

3

Learning to Self-Assess then Self-Assessing to Learn: Integrating the RTOP into Mathematics Teacher Preparation

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According to contemporary pedagogical approaches it is expected that mathematics teachers assume reform-based instruction. It is generally recommended that reform-based instruction focus on the inquiry and discovery of mathematical ideas; teachers should create and teach lessons encouraging students to investigate mathematical sense making through creative approaches (Lee, 2007; Warfield, Wood & Lehman, 2005; Watt, 2005; Smith, Desimone & Ueno, 2005). The focal point is open-ended, problem-solving investigations that deal with multi-layered mathematical ideas (Maccini & Gagnon, 2000). Mathematics is more than computation, rather "doing mathematics requires students to examine, explore, communicate, conjecture, reason and argue" (Cooney, Sanchez & Ice, 2001, p. 10). Mathematics has changed; the quick computation of numbers is not a measure of success anymore but instead there is an emphasis on conceptual understanding of ideas structured through inquiry-based lessons (Smith, Desimone & Ueno, 2005).

While describing reform-based mathematics, it is necessary to reconsider assessment practices as well. Teachers should pay more attention to discourse in mathematics including examining students' ideas and reasoning, rather than just the mechanics of mathematics (Cohen & Hill, 2000). Alternative types of assessments beyond tests and quizzes must become more common in our mathematics classrooms (Ohlsen, 2007) and no single assessment can provide a complete picture of what a student knows (Hong & Ehrensberger, 2007). Assessment approaches that are inadequately linked to reform-based teaching practices will most likely lead to inappropriate information in regards to problem solving and critical inquiry. If there is an adoption of reform-based inquiry teaching rather than traditional lecturing, then there must also be an alignment with alternative assessment practices (Cooney, Sanchez & Ice, 2001). In their research on the use of alternative assessments in mathematics, Flexer and Gerstern (1993) found that teachers who used alternative assessments made changes in their curriculum, making sure that their instruction correlated with their assessment practices. Hence, alternative assessments can guide inquiry-based learning. Alternative types of assessment can range from how the students are assessed to how the teacher self-assesses in terms of her/his presented inquiry-based investigations of mathematical ideas. While student assessment results are important to use in determining mathematical paths, whether or not the lesson contained ideas that are considered valuable and significant for reformed-based instruction should also be evaluated to help guide the teacher's instruction.

In this paper, the reform-based teaching model in mathematics is examined along with ways to support it using alternative assessment types. While alternative assessment types are usually analyzed in terms of what the students accomplished (or failed to accomplish), it is important to also examine the teacher. Lessons and practices must also be assessed to establish whether or not the instruction provided appropriate opportunities for inquiry and problem-based learning. Continuous professional learning creates a basis for a quality teaching (Lee, 2007; Morris, 2006), while self-assessment is necessary to determine the direction one should take in relation to the learning process (Scherer & Steinbring, 2006). A framework is provided to integrate alternative teacher assessments into a program of study for future California secondary mathematics teachers. It is hoped that this alternative assessment can be used as a tool for lifelong learning and growth by influencing and guiding future lessons.

Teaching in Mathematics

Koehler & Grouws (1992) describe five distinct approaches to mathematics teaching and learning: constructivism, cognitively guided instruction, expert-novice paradigm, sociological or epistemological view, and mathematical content view. With each of these approaches, students are not passive absorbents of information. Instead, they are active participants in the acquisition of knowledge and strategies, while teachers are viewed as informed and reflective decision makers (Ticha & Hospesova, 2006). Morrell, Flick and Wainwright (2004) describe reform-based practices as aligning with the constructivist view of teaching and learning. Constructivist practices are based on the facilitation of discourse among students, supporting inquiry and problem solving, and assisting students through reflections on their own learning (Wheatley, 1992).

Since the National Council of Teachers of Mathematics (NCTM) began recommending instructional changes in 1989 there has been a transformation concerning how students are taught mathematics (NCTM, 1989; Sawada et al., 2002; Watt, 2005; Spillane & Zeuli, 1999). Instructional programs that align with NCTM recommendations are generally thought of as reformed-based mathematics (Gabriele & Joram, 2007). Reform-based mathematics should encourage students to think and evaluate the mathematics they explore, rationalizing the processes to make sense of the investigations (NCTM, 2000).

Additionally, mathematics should not be undertaken in isolation. Jaworski (2006) describes inquiry teaching, taking into consideration the perspective of critical alignment, which is achieved through communities of inquiry involving students, teachers and educators, and in which all participants are learners. Inquiry-based activities encompass a broad spectrum, ranging from teacher-directed inquiry which is structured and guided to open inquiry which is student-directed (Olson & Loucks-Horsley, 2000). According to Zion, Cohen and Amir (2007) teachers play a critical role in open inquiry learning by facilitating, focusing, challenging and encouraging students to engage in appropriate activities.

Standards in Mathematics

Standards are designed to guide the instruction of mathematics and determine the direction and ways in which concepts are investigated. The NCTM (2000) created *Principles and Standards for School Mathematics* to help guide the national implementation of reformed-based, inquiry mathematics. According to the NCTM (2000) *Standards*, there are six basic principles to a comprehensive mathematics program including: Equity, Curriculum, Teaching, Learning, Assessment, and Technology. The Assessment Principle concentrates on using information to inform and guide teachers' instruction; "it should be done *for* students, to guide and enhance their learning" (NCTM, 2000, p. 22). Assessment needs to go beyond an end of the unit test; it should be ongoing and come from multiple sources to guide the path of instruction (Hong & Ehrensberger, 2007) and should align with the learning goals of the curriculum (Whittaker & Young, 2002).

While assessment is shown to be an important element with national considerations, it is also valuable according to the *Mathematics Framework for California Public Schools Kindergarten through Grade Twelve* (California Department of Education (CDE), 2000). There are three purposes of assessment including: Entry-Level Assessment, Progress Monitoring and Summative

Evaluation. Of these, Progress Monitoring aligns most with assessment that takes place on a regular basis in relation to the daily mathematics lessons. Progress Monitoring allows the teachers to make adjustments to better meet the needs of the students and is critically important for making well-informed decisions about the direction of the lessons. Deciding what is known and what still needs to be known in connection with the California standards is a valuable element of progress monitoring (CDE, 2000). According to Cohen and Hill (2000) teachers' practices improve when they consistently assess their students' progress and make adjustment to the curriculum based on the findings.

While both the NCTM and CDE emphasize assessment and teachers' examinations of what their students know and understand, it is also important to assess one's own teaching (Smith, Desimone & Ueno, 2005). Assessment should go beyond what it means for the students to know and understand, and must also consider what the presented lesson achieved (or neglected) in terms of supporting students' growth and development of mathematics. Other principles address this concern.

The Teaching Principle (NCTM, 2000) addresses the need for teachers to grow and improve on their ability to teach mathematics including self-reflection. "Opportunities to reflect on and refine instructional practice—during class and outside class, alone and with others—are crucial to the vision of school mathematics" (NCTM, 2000, p. 19). There is a need for reflection and analysis (preferably with collaboration) of one's own lessons while teachers should also have a deeper understanding of their own instruction (Ticha & Hospesova, 2006). If mathematical learning is viewed as a collaborative effort amongst a community of learners (Jaworski, 2006), then assessment should take place in relation to all the participants including the teacher.

Along these same lines, the *Mathematics Framework for California* (CDE, 2000) addresses the need to increase "teachers' mathematical knowledge and ability to teach the subject" (p. 217). There is a need to provide California mathematics teachers with opportunities to grow and expand on their mathematical understanding. According to Cohen and Hill (2000), opportunities for California teachers to learn about mathematical content and how to teach it are very important for successful growth in mathematics. Teachers' practices have a large influence on how students develop mathematically (Cohen & Hill, 2000) and evaluation of instructional procedures can have powerful influence on students' mathematical understanding (Jaworski, 1998). Both the NCTM and the CDE place emphasis on teachers as evolving, growing and developing into more qualified life-long learners capable of adjusting to meet the mathematical needs of their students.

Assessment in Mathematics

The term assessment refers to all activities undertaken by teachers and their students that provide information to be used as feedback to modify teaching and learning activities (Black & Wiliam, 1998). According to Niss (1993), assessment in mathematics addresses the outcome of mathematics teaching at the student's level and focuses on estimating mathematical capability, performance and achievement. According to themes present in a literature review (Ruthven, 1994), authentic mathematical assessment tasks should: (1) have plausibility beyond the school system, (2) make sense and engage students, (3) allow students to use their insights and ideas, (4) provide the opportunity for students to form questions and investigations, (5) have the ability to be undertaken in multiple ways and (6) allow the use of various resources. Wheatley (1992) describes assessment as a way for teachers to learn about their students' mathematical constructions; teachers should take the time to gather information about how their students rationalize and explain their mathematical understanding. Black and Wiliam (1998) emphasize that the main purpose of assessment is the improvement of learning. Ohlsen (2007) describes four purposes of assessment including: grading, identification of students' special needs, motivation and monitoring the effectiveness of a lesson.

Within a traditional setting, assessment is solely the quickness and correctness the students demonstrate in computing numbers. According to Pegg (2003), assessment in mathematics is dominated by a focus on content (facts), skills (computational techniques) and the ability of learners to reproduce these on demand. This has a negative effect on innovations and developments within the mathematics curriculum. In a traditional setting, mathematics assessment may only take the form of quizzes and tests (Kirtman, 2002; Henke, Chen & Goldman, 1999). Additionally, even in NCTM aligned classrooms there is a high reliance on evaluating student learning solely through the use of tests and quizzes (Ohlsen, 2007). Watt (2005) argues that traditional mathematics tests do not provide a valid measure of student ability; however, teachers are opposed to implementing alternative assessments due to their subjectivity. The perception is that alternative assessments are too time consuming, and there is a potential for grading controversies, such as lack of uniformity and fairness (Cooney, Sanchez & Ice, 2001).

While teachers may be apprehensive about alternative types of assessment, these assessments offer more insight as to what the students know above and beyond just a unit test. Student assessment in mathematics must come from multiple sources to provide a more holistic picture of the student's growth and understanding (Hong & Ehrensberger, 2007). According to Watt (2005) too much reliance on one form of assessment does not account for different learning styles. In addition Watt (2005) believes that using only one form of assessment will bias the students toward one kind of learning, preventing a rounded mathematics curriculum.

Assessment of students' mathematics learning has a large impact on the way mathematics is understood in schools (NCTM, 1995). NCTM (2000) emphasizes how important it is for assessment to be more than a test at the end of a unit, but instead should be used to inform and guide teachers as they make instructional decisions about mathematical learning. The *Mathematics Framework for California* (CDE, 2000) recommends that teachers be knowledgeable about the various forms of assessment available.

Assessment in reformed mathematics takes many different shapes and primarily focuses on an analysis of students' learning and achievement (Ohlsen, 2007). Some alternative forms of assessment for reform-based mathematics include short answer questions, essays, performances, oral presentations, demonstrations, exhibitions, and portfolios (Laboratory Network Program, 1994). Bahr (2006) describes different techniques to support alternative assessments such as open-ended questions, journals, performance-based assessments, observations, and constructed-response tasks. Gearhart and Saxe (2004) provide evidence that students benefit when their teachers who use inquiry methods engage in ongoing assessment of understanding.

While most assessment focuses on the student and his/her explanation and understanding of curricular ideas, it is also important to consider how reformed the lesson was in providing opportunities to discover mathematics (Sawada et al., 2002). Whether or not the students had the opportunity to investigate and discover mathematics is directly related to how the lesson was presented (Olsen & Kirtman, 2002). While assessment usually focuses on the student, measuring and analyzing instructional practices play an important role as well (Smith, Desimone & Ueno, 2005). Lessons that are presented in a traditional manner emphasizing lecture, rather than the imitation of procedures, will not generate reformed learning (Wainwright, Morrell, Flick & Schepige, 2004; Becker & Jacob, 2000). So, how do beginning teachers learn to assess their teaching? How do they recognize whether or not their reformed lesson was successfully structured to bring about optimum student success? Sensible assessments that align with the curriculum goals are very important (Whittaker & Young, 2002). The Reformed Teaching Observation Protocol (RTOP) offers a solution as it allows teachers to evaluate lessons to determine how well they aligned with reformed-based mathematics.

RTOP

The RTOP is a tool used to guide teachers in the self assessment of their own instruction in relation to defining and measuring reform (Center for Research on Education in Science, Mathematics, Engineering and Technology (CRESMET), 2004; Sawada et al., 2002). CRESMET (2007, p. 1) developed the protocol to support its overall mission to "engage in research that creates new knowledge and tools that improve the quality of STEM teaching and learning in real-world classrooms." The emphasis is on reform-based teaching that aligns with what a constructivist-based classroom should highlight in relation to inquiry- and standards-based student centered teaching and learning (Piburn & Sawada, 2000; Sawada et al., 2002). The RTOP is a reflective instructional guide that can be electronically downloaded (available at http://cresmet.asu.edu/prods/rtop.shtml) or used in paper form.

The RTOP consists of 25 individual components divided into three different subsets: *Lesson Design and Implementation*, *Content* and *Classroom Culture* (Piburn & Sawada, 2000; Sawada et al., 2002). *Lesson Design and Implementation* relates to the format of a reform-based lesson including accessing prior knowledge, engaging the students as learners to investigate a problem and determining the direction of the lesson based on the students' ideas. *Content* is divided into two sections, the first focusing on the content and the second emphasizing the inquiry process. *Classroom Culture* focuses on the culture of the classroom in relation to two areas, communicative interactions and student/teacher relationships (MacIsaac & Falconer, 2002; Sawada et al., 2002).

Within each of the subsets, there are several different statements ranked on a five point Likert scale, indicating whether the concept never occurred (0) or was very descriptive (4) in relation to the taught lesson. The RTOP was designed to span grade levels from kindergarten to university levels (MacIsaac, Sawada & Falconer, 2001). Summing the scores will provide the degree of reform (MacIsaac & Falconer, 2002). For example, a score of 20 or less indicates a traditional lesson while scores of 65 to 99 indicate reform-based teaching.

In terms of development, the RTOP was created with input from numerous researchers and developers. There was an emphasis on mathematics and science teaching rooted in reform-based standards and ideas (Piburn & Sawada, 2000). The RTOP was systematically evaluated and discussed, with changes made throughout its development. After the protocol was finalized, it was tested in classrooms resulting in a very high overall estimated reliability of 0.954 (Piburn & Sawada, 2000; Sawada et al., 2002). . The test results suggest that it is possible to achieve very high inter-rater reliabilities using RTOP. Detailed analysis of the RTOP suggests that the instrument is sufficiently able to measure reformed teaching (Piburn & Sawada, 2000).

Using the RTOP requires some training and understanding of its structure and use. With proper training on its use, the overall evaluation of the RTOP indicates that it is a reliable instrument to evaluate reformed teaching in mathematics and science classrooms (Piburn & Sawada, 2000). With such positive reliability, this instrument can be used as part of a comprehensive program to guide preservice secondary mathematics teachers. If the program of study emphasizes teaching and learning based on NCTM (2000) *Standards*, then it is important for the prospective teachers to have the ability to reflect and assess whether or not their lessons (as taught) aligned with the appropriate reform-based framework.

Quality teaching is more than just planned and organized activities; it must also entail activities that are reflected upon and critically analyzed (Scherer & Steinbring, 2006). Darling-Hammond and Snyder (2000) write that instead of implementing set routines, teachers need to implement activities that help them to evaluate teaching situations and develop effective teaching responses. This call for teachers who are evaluators and developers of effective teaching practices could be answered by the implementation of teachers' reflective practices. Teacher change can occur through reflection on experiences (Ross & Bruce, 2007).

Garcia, Sanchez, and Escudero (2006) write that reflection implies the conscious engagement of an individual in practice. The authors presented ways in which reflection-on-action was used among a group of mathematics teacher educators and researchers to improve their professional development by connecting theory and practice. When reflecting on the relationship between theory and practice, researchers found new theoretical perspectives. At the same time mathematics teacher educators learned how to make use of theory. In addition, practice can be a context of study and research. Jaworski (1994) describes how theory, research and reflective practice interconnected with her own learning through her classroom research. In her later studies, Jaworski (2006) further explores a concept of "inquiry" in mathematics learning and teaching. She emphasizes that inquiry "as a tool" and inquiry "as a way of being" are important concepts in reflexive developmental processes in which inquiry practice leads to better understandings including the development of theory. The individualized assessment of the RTOP results may provide the necessary feedback to reflect upon implemented lessons influencing the development of future lessons and curriculum (Sawada et al., 2002).

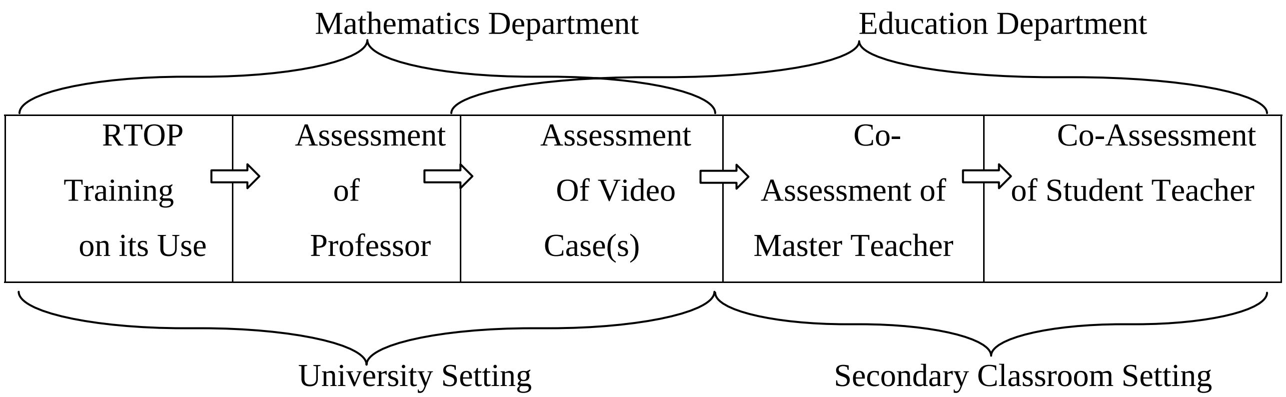
Integrating the RTOP into the Mathematics Teachers' Curriculum

A comprehensive program to integrate the use of the RTOP in California mathematics classrooms is recommended to enhance the learning and understanding of teaching mathematics at the secondary level. In order to integrate the RTOP, strategic planning for long term use is suggested in order to support mathematics teachers' awareness of the strengths and limitations of their lessons. It is the hope that such an analysis can inform future curricular decisions to help improve how reform-based mathematics is taught in the classroom.

Assessment should be part of the classroom routine (NCTM, 2000) and the RTOP provides a framework to evaluate mathematics teaching on a regular basis. Teachers need to continually seek improvement to be more effective (Lee, 2007) and the RTOP provides specific feedback on what component(s) should be enriched. How can the RTOP be integrated into a mathematics teacher preparation program? When teachers complete their coursework and begin to teach independently, it is important that they know how to evaluate their own teaching. This evaluation can potentially lead to lifelong learning and growth, assisting in the successful implementation of more reform-based teaching centered on feedback from implemented lessons (Gabriele & Joram, 2007). For example, if the RTOP scores are relatively low, it signals that changes must be made in terms of the design and delivery of the curriculum in order to better align with reform-based mathematics teaching.

To successfully build the RTOP into the curriculum, there needs to be multiple experiences with the framework; it cannot happen once as transformations in teaching generally take place over time (Gabriele & Joram, 2007). There needs to be a seamless integration throughout numerous mathematics content and methodology courses. Proposed is a framework to integrate the RTOP into the core subject-matter courses within the mathematics department, along with the credentialing courses within the education department, see Figure 1 for a brief overview. The RTOP has been shown to be a supportive, reflective device and can guide preservice teachers in monitoring and understanding their own teaching practice (MacIsaac, Sawada & Falconer, 2001). And according to Piburn and Sawada (2000), "RTOP scores predict improved student learning in mathematics and science classrooms at all levels" (p. 24). If it is used as a tool throughout the core course work and credentialing experience, it may strengthen the way students are taught mathematics while improving the prospective teachers' understanding of quality mathematics instruction.

Figure 1. Displayed are potential steps for the integration of the RTOP into a credentialing program for mathematics teachers.



First and foremost, programs must start with training on how to use the RTOP. In order to support the use of the RTOP, there are some protocols available online at the training site (http://cresmet.asu.edu/prods/rtop.shtml). At this site, the RTOP reference manual (Piburn & Sawada, 2000), training guide, and forms, along with a link to a training website can be found (CRESMET, 2004). The training guide provides specific information on how to use the RTOP and is designed to assist in the implementation of the protocol (Piburn & Sawada, 2000). This framework can be used in the university setting to introduce the use of the RTOP along with a rationale for its application.

After training, the RTOP could then be employed to evaluate teaching. The professor of the course could model a lesson and ask his/her students to assess the lesson through the paper or electronic version of the framework. Groups could unite and discuss the discrepancies of the scores and try to come up with a consensus (MacIsaac, Sawada & Falconer, 2002). Both examples and non-examples of reformed-based mathematics could be modeled by the professor; the professor may want to teach a direct instruction mathematics lesson followed by a reform-based mathematics lesson and compare the two results. Analysis of the differences can be further supported through classroom discussion along with analysis of the pedagogical structure of the lessons.

After several university lessons have been evaluated, the professor could progress to secondary classrooms. It seems that it would be very difficult to have all of the students in a class observe the same teacher and lesson due to space issues and scheduling difficulties. With the implementation of video cases, it is possible for all of the students to observe and evaluate the same classroom. Monroe-Baillargeon (2002) described video cases as offering an experience for which students or teachers can evaluate teaching issues, dilemmas and opportunities. Video cases can help preservice teachers connect theory and practice while stimulating thought (Shulman, 1992) and they can provide a context to promote analysis (Santagata, Zannoni & Stigler, 2007). Examining lessons in terms of the effects on learning is a very useful skill needed for teaching (Morris, 2006).

While textbooks sometimes have video cases to support course readings, there are also some available online for free. For example, the Best Practices site (http://pt3.ed.asu.edu/bestpractices/) showcases mathematics, science, and technology lessons from kindergarten to junior college (Kurz, Llama & Savenye, 2005). These videos can be used as cases for evaluation using the RTOP framework, and then a group discussion could be used to evaluate teaching observations, issues and concerns. Video cases have been used as part of the RTOP experience and have generated positive results (MacIsaac, Sawada & Falconer, 2001). In addition, video cases have been used with encouraging results to guide preservice teachers' analysis of lessons (Morris, 2006; Santagata, Zannoni & Stigler, 2007; Ticha & Hospesova, 2006).

Through their program of study, many preservice mathematics teachers experience half-day and full-day student teaching as part of their credential preparation. In an ideal situation, the preservice teachers would be placed in classrooms that emphasize reform-based instruction and inquiry-based learning. If preservice teachers were placed in this setting, then the master teacher, preservice teacher and university supervisor could assess teaching together using the RTOP. Through a community, a cooperative spirit could potentially emerge to assist in the development of more reform-based teaching as pairing newer teachers with veteran teachers can be beneficial (Olsen & Kirtman, 2002) to growth. By evaluating the master teacher's taught lessons, this cooperative community can better gauge how well the lessons aligned with what is necessary to yield optimum learning in mathematics.

As teachers become more and more experienced with the evaluation of others, it is important to provide them with opportunities to assess how aligned their own lessons were in terms of reform-based teaching. According to Scherer and Steinbring (2006), improving one's own teaching through everyday mathematics activities is helpful for improving mathematics learning, and reflecting collaboratively can also be very fruitful. The final step would involve preservice teachers' evaluation of their own instruction while in their student teaching placements, with the cooperative support of master teachers and university supervisors, while also videotaping their lessons for further evaluation (Lee, 2007). The university evaluation forms used by the supervisors could align with the RTOP structure. This would allow for a more cohesive transition into the student teaching environment through the familiarity of the evaluation structure. Newer California teachers usually focus on whether their lessons accomplished procedural aspects along with an analysis on how the alignment of the curriculum related to standardized tests in mathematics; they were unable to look at reform issues (Drake, 2002). Perhaps the RTOP can take teachers beyond this surface level analysis.

There are several areas of concern if integrating the RTOP into the program of study for preservice mathematics teachers in California. First, both the mathematics and education departments must work as a team to integrate such a program. Generally the mathematics departments are responsible for content courses while the education departments are responsible for methodology courses and the supervising of student teachers (Tobias, 1999). In order for this program to be successful, both these departments must come together and unite to integrate the RTOP into the curriculum.

The second area of concern is the use of the RTOP in the student teaching environment. While some of the mathematics classes in California adhere to a reform-based structure, there are others that do not (Drake, 2002). Currently, the *Mathematics Framework for California* (CDE, 2000) does not embrace a complete reform-based structure despite compelling reasons to do so (Becker & Jacob, 2000; Drake 2002). However, *Principles and Standards for School Mathematics* (NCTM, 2000) describes reformed-based mathematics as valuable to bring about mathematical thought and inquiry. So even though there is some pressure for California classrooms to emphasize drill and rote memorization (Becker & Jacob, 2000; Manzo, 2001) every effort must be made to find properly aligned placements that support reform-based investigations.

Along these same lines, while the *Mathematics Framework for California* (CDE, 2000) does support some use of reform-based ideas, there is still an overriding presence of surface level, drill learning (Becker & Jacob, 2000; Manzo, 2001). And even though some districts, schools and teachers do adhere to reform-based instruction in mathematics, there will always be a great importance on standardized test results as long as money is tied to student performance. Currently, the state standardized test has a predominance of questions dealing with computation and basic procedural skills (Becker & Jacob, 2000; Manzo, 2001; Whittaker & Young, 2002). Procedural instruction is also very common nationally in mathematics (Smith, Desimone & Ueno, 2005; Wainwright et al., 2004). With this emphasis, it may be hard for teachers to be devoted to reform-based teaching and alternative assessment techniques (Whittaker & Young, 2002). Even though there are challenging obstacles to implementing the RTOP as part of an assessment change in California, it is a step that may lead to better teaching and learning.

Implications

According to the NCTM (2000), "assessment must become a major focus in teacher preparation" (p. 24). Yet assessment must go beyond evaluating students' understanding and must also examine how teachers guide their students to discover (Ticha & Hospesova, 2006). Effective teaching must go beyond pedagogical strategies and must also require teachers to continually seek improvement (Lee, 2007). The RTOP provides a foundation for teachers' self-assessment contributing to their lifelong education and evolvement to guide self-improvement. When introduced and implemented into teacher preparation programs, it has the potential to guide how teachers' interpret their own reform-based instruction. It is hoped that once teachers experience the positive benefits of the RTOP, they will then evaluate and assess their own teaching beyond procedural aspects and standardized test preparation analysis.

In a study by Olsen and Kirtman (2002) that examined restructuring in California schools, it was found that experiences influence the willingness teachers have to implement change within the classroom. For example, teachers who had positive experiences with rubric-based assessments were more likely to try new, reform-based practices that encourage rubrics. This relates to the RTOP integration. It can be expected then that teachers who have had positive experiences throughout their teacher preparation courses in college with reformed-based assessment of teaching maybe more likely to view this approach as valuable and then support its use if given the opportunity.

According to Gabrielle and Joram (2007), newer teachers who evaluated their own teaching tended to focus on meeting curriculum goals while lacking precise explanations and evaluations of student thinking as a means of successful teaching. Smith, Desimone and Ueno (2005) found that novice teachers were more likely to focus on procedural instruction in mathematics than conceptual development. In order to improve reform strategies, college faculty members must incorporate tools and techniques to guide practices (Wainwright et al., 2004). Because of its structure, the RTOP may allow novice teachers to go beyond this shallow explanation of successful teaching. The RTOP has been researched, and the evaluation of lessons supported by interpretive commentary can bring about reflective change in teaching (MacIsaac, Sawada & Falconer, 2001). If this structure becomes part of a mathematics teacher's collection of tools available for assessment, then perhaps lessons will become more successful. The RTOP provides a tool to assess one's own teaching and lessons focusing solely on reform (MacIsaac, Sawada & Falconer, 2001), providing direct feedback to the teacher on where the lesson was strong and where it was inadequate. If the RTOP assessment results are used correctly, the information could then be used to create, adjust, or alter upcoming lessons on a daily or weekly basis; lessons that are unsuccessful or ineffective can be removed in upcoming years and replaced with more reform-based investigations.

Mathematics should not be taught in isolation. NCTM (2000) suggests that professional development focus attention on creating local communities that promote the practice of shared investigations grounded in teachers' work. There is a social component of inquiry learning that is highly emphasized. Jaworski (2006) argues that mathematics itself takes place in a social setting based on the fact that it has developed over the years through human thought and interactions which are characteristics of social settings. With this in mind, it is important to create a social environment for integrating the RTOP into practice with novice teachers and their student teaching community. Teachers who integrate the RTOP into their development and continue to create and implement reform-based practices can serve as leaders for future preservice teachers. Furthermore, constructivist teachers have active roles and networks within their professional community, potentially leading others in their development (Judson & Lawson, 2007). So in terms of lifelong learning, the RTOP can play a role in helping to create more reform-based practices throughout the community.

While there is no single magic formula that will work to improve all teaching and learning in mathematics, the RTOP offers hope. It can improve the way children are taught mathematics when used as part of a comprehensive, reform-based mathematics program by guiding novice teachers and helping them evaluate their own teaching. With the push for multiple types of assessment practices (NCTM, 2000;Whittaker & Young, 2002; Ohlsen, 2007), the RTOP provides a new, innovative approach to assess one's own practices and is one step in that ever important improvement formula.

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4

Preparing Secondary Teachers of Mathematics with and for Democratic Practice[1]

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> *If children are taught mathematics well, it will teach them much of the freedom, skills, and of course the disciplines of expression, dissent and tolerance that democracy needs to succeed. If, on the other hand, they are taught mathematics as if it has no room for independence; as if they must never question, doubt, or disagree; and if we therefore fail to teach them to respect and value those who have different ideas – or wrong ideas – or even no ideas at all (as Socrates insisted he had none) – then we can do more than damage their mathematics. For this kind of mathematics teaching destroys democracy. (Hannaford, 1998, p. 186)*

It has been argued that in the twenty-first century United States access to mathematics, especially algebra, is a civil right no less important than the right to vote (Moses & Cobb, 2001). Among the reasons that students, disproportionately those who are labeled as poor and minority, turn away from and/or are rejected by mathematics include perceptions of ability by those in authority, cultural discontinuity between learning preferences and instructional practices, inequities in mathematics coursework placement, and the lowered expectations of teachers, parents, or society (Gutierrez, 2007; Malloy & Malloy, 1998; Plata, Masten, & Trusty, 1999; Rousseau & Tate, 2003; Volmink, 1994)—practices that are inherently undemocratic.

Indeed, observational studies of mathematics classrooms in the United States have found them to be dominated by teacher-talk aimed at transmitting fixed knowledge to students and assessing how well it was received (Stigler & Hiebert, 1997; Stodolsky, 1988; Weiss, Pasley, Smith, Banilower, & Heck, 2003), practices political scientist and philosopher Amy Gutmann (1999) refers to as a disciplinary approach through which "teachers assert their authority, first to produce order, and then to funnel a body of knowledge into students" (p. 89). Such a disciplinary approach "situates mathematics as *a priori* knowledge, based on objective reason alone, without taking into account the experiences students bring to mathematics or the meaning they make of what is learned" (Ellis & Berry, 2005, p. 8). When this is the exclusive manner in

[1] This paper builds on a presentation by Ellis, M. W., & Malloy, C. E. (2007).

which students experience mathematics learning, it is not surprising that so many find themselves alienated from mathematics and find the subject to have little relevance or connection to their lives.

In contrast to the norm, there are several recent examples of teacher practices that help students—in particular those historically underserved by schooling—develop mathematical content knowledge in ways that also promote more democratic forms of participation both within the classroom and in the school and local community (Gutstein, 2003; Gutstein & Peterson, 2005; Turner & Font Strawhun, 2007; Vithal, 1999). Such work offers possibilities for reformed practice and poses a challenge to mathematics educators to consider how to prepare teachers who are capable of creating more democratic learning environments.

It is imperative that in preparing new teachers of mathematics attention is given to developing both the tools and the perspectives necessary for them to be well-equipped for the task of creating learning environments that allow a broader range of students access to making sense of and taking ownership of mathematical knowledge, ultimately "creating conditions for young people to become active participants in changing society" (Gutstein, 2006, p. 4). This task is made more challenging given the fact that most pre-service teachers (PSTs) have experienced "mathematics classes as an immutable sequence beginning with checking homework, asking questions about the homework, watching the teacher demonstrate how to do new problems, then receiving an assignment to work problems similar to those demonstrated" (Sowder, 2007, p. 199; see also Weiss et al., 2003).

This article will first delineate the notion of democratic education and how it can be applied in the mathematics classroom, then offer examples from the author's mathematics methods course of strategies intended to prepare PSTs both with and for democratic practice, and finally share reactions PSTs have had to this work. It is hoped that these ideas will serve to stimulate thought about how, in addition to focusing on teaching skills such as lesson planning and assessment design, the methods course can stimulate critical reflection about the role and responsibility of mathematics teachers in preparing students for active participation in democratic communities.

Democratic Education

The literature on democratic education identifies characteristics of democratic classrooms as including: a) open exchange of ideas; b) belief in the ability, individually and collectively, to solve problems; c) use of reflection and analysis to examine claims; d) equal participation in decision-making; and e) concern for problems of others and the community (Beane & Apple, 1995; Malloy, 2002; Pearl and Knight, 1999; Skovsmose, 1998). Importantly, these do not represent a prescribed set of practices but rather "an 'idealized' set of values that we must live and that must guide our life" (Beane & Apple, p. 7). What is more, it is not argued here that schools can or should function as entirely democratic societies; as Gutmann (1999) points out, schools must ultimately "balance the participatory [democratic] and disciplinary purposes of education, leaving some significant educational decisions—such as the content of the curriculum and the standards for promotion—largely (but often not entirely) to the determination of teachers and administrators" (p. 93). As such, efforts toward more democratic schooling practices will always be a work in progress.

Framing the characteristics of democratic education in relation to the mathematics classroom, teachers are called on to create environments in which students a) communicate with one another freely with and about mathematics; b) have confidence in their individual and collective ability to solve mathematics problems; c) justify and revise mathematical claims; d) have a voice in decision-making and authority; and e) learn to use mathematics to better understand important issues of their school and community.

The importance of a democratic or participatory approach to mathematics education is made clear by mathematics educator Colin Hannaford (1998) who argues that mathematics learning has the power to develop in students the sorts of habits necessary for their democratic participation in society—reasoned expression, justified dissent, and tolerance for multiple perspectives. Mathematics is an ideal subject through which students can advance their abilities to reason, discuss, and understand for and among themselves. However, when students experience mathematics as rote, disconnected, and without meaning, this "destroys democracy" (Hannaford, p. 186) by cultivating a mindset that passively accepts that which often makes no sense at all.

The Preparation of Teachers for Democratic Mathematics Classrooms

It is within teacher preparation coursework that the traditional and often undemocratic habits of teaching and learning mathematics can be challenged and disrupted. Through more participatory and democratic practices enacted within the mathematics methods course, we can "open opportunities for discussion and debate, implement strategies that develop student authority, and draw connections between mathematics and community and societal issues" (Ellis & Malloy, 2007). What follows are some of the strategies that have been used to create experiences for PSTs that touch on each of the five characteristics of democratic mathematics classrooms delineated above. While not an exhaustive list, it is representative of the foundation on which other course assignments and activities rest.

Challenging the Authority of Mathematics

One of the most critical beliefs about mathematics that must be challenged in order to move toward more democratic practice is that mathematics is a static collection of rules and procedures to be mastered one at a time by rote without much concern for personal connection or understanding (Ellis, 2007; Stodolsky, 1988; Walkerdine, 1998; Walshaw, 2002). In order for students to have a connection to learning it must be experienced as personal and relevant. The Problem of the Day and Mathematics Autobiography activities described below represent efforts toward disrupting PSTs' preconceived ideas about the nature of mathematics and how it is learned by providing opportunities to revisiting understandings, share alternative perspectives to solving mathematical problems, and justify and challenge each others' mathematical claims.

Problem of the day. Given at the start of each weekly class session, these problems require PSTs to generate multiple solution paths and/or to generate justifications for taken-for-granted mathematics procedures. Collaborative discussions expose students to alternative ways of thinking and help them strengthen their skill in expressing ideas and justifying their mathematical reasoning. For example, students might be asked to develop two ways to prove why the area of a trapezoid is equal to $\frac{1}{2}(b_1 + b_2)(h)$. They are given time to work individually, then move into pairs or groups to further discuss their ideas and justify their solutions. In reviewing the problems students are asked to present their thinking—erroneous and correct—for the whole class to consider. For many it is the first time they have had such discussions in the context of a mathematics (methods) class. Ultimately, what comes of these investigations is an awareness of the flexibility of mathematical reasoning, the ability to develop insights into mathematics through listening to others, and the opportunities this presents to build upon one's own understanding of mathematics.

Mathematics autobiography. In a study of over 300 college students' mathematical autobiographies, Hauk (2005) found a predominant view of mathematics as external, authoritative knowledge that resides in teachers and textbooks. Such ideas are powerful to interrogate at the start of a methods course for PSTs of mathematics. The Mathematics Autobiography assignment precipitates reflections on and discussions about students' prior experiences with and perceptions of mathematics. Specifically, PSTs describe in writing their experiences with mathematics throughout their lives both in and out of school, positive and negative. They are told to include their earliest recollection of doing math, instances of people who influenced their thinking about mathematics, and experiences that shaped their perception of their ability in mathematics. When these are brought to class students first are given time to share their narrative with a partner. The whole class conversation that follows brings to light the varied experiences students have had with mathematics and leads to questions about traditional practices of schooling that position mathematical knowledge in ways that generate success for a few students and struggles for most others.

Building Social Awareness and Student Authority

Just as learners of mathematics often perceive themselves as apart from the content being learned (Ellis, 2007), PSTs may see themselves as apart from the knowledge of teaching. In both cases, the danger is that students come to view such knowledge as outside of themselves, something to be understood at a distance rather than something to make their own. Democratic education challenges this positioning by placing knowledge within the community, within reach of everyone, and open to reasoned critique. Inasmuch as we want learners of mathematics to become doers of mathematics (National Council of Teachers of Mathematics, 2000), we want pre-service teachers of mathematics to become active agents of education. This requires they learn to access and interpret research relevant to teaching mathematics, to understand and communicate with the communities in which they work, and to take action to effect change within various spheres of influence (e.g., classroom, department, school, and district). The Reading Facilitation and School Profile assignments are aimed at helping PSTs to develop these abilities and inclinations.

Reading facilitation. This assignment requires that PSTs work in pairs to develop a set of discussion questions (in collaboration with the instructor) that they then use to facilitate a class discussion for an assigned reading (e.g., journal article). Their role in preparing the questions and leading the discussion is peer-evaluated using a rubric (see Appendix 1). The democratic power of this process is two-fold. First, PSTs are challenged to make sense of knowledge that is "out there" in a journal article and to reflect on it in terms of their own experiences and perspectives. Second, the roles of student and instructor are upended during the in class facilitation, giving those leading the discussion the opportunity to demonstrate their understanding of the reading and their skills in respectfully drawing out the ideas of others.

School profile. Pre-service teachers are typically assigned to student teach at a school site about which they have little knowledge. This assignment requires them to research the history of the schools to which they are assigned so they can learn about the communities of which they will become members. They collect information by reviewing online public databases, talking to school employees (teachers and staff members), looking at school yearbooks, and meeting with students and parents. They investigate demographic changes in population over time, challenges and successes of the school and school community, perceptions of the school from the inside and outside, and aspirations of teachers, students, and parents. In discussing their findings in class, PSTs are challenged to consider the sorts of skills, knowledge, and understandings they will need to develop in order to become effective teachers in these schools and communities.

Reactions of Pre-Service Teachers

At the end of each semester PSTs are asked to write a reflection about their experiences in the methods course in which they think about what they have learned about becoming a teacher of mathematics. By sharing some of their comments, the intention is to offer an informal glimpse into how the course is received; it is acknowledged that this is not a formal exploration of how it impacts PSTs. Reading through their comments, it is not surprising that, as pre-service teachers, much of their attention is focused on concerns with their own mathematical development (Conway & Clark, 2003). Their responses have consistently demonstrated a growing awareness of the need to continually extend understandings of mathematics that were gained largely in traditionally structured learning environments.

> *My mathematical knowledge in the past I would have to say was mostly rules and operations. I had no idea there was so much behind what I am teaching. (Ms. Fontaine)*

> *I was taught math by the traditional direct teaching method. I was able to learn math by trying a problem for homework, checking the answer in the back of my textbook, and then working from the answer to the problem if my answer was incorrect the first time. This*

sort of math education does not make me necessarily ready to teach mathematical concepts. I had this realization about my mathematical abilities during the course of the math methods class. (Mr. Collins)

In addition, many PSTs have reflected on how they will impact the students in their future classrooms, especially in light of a tradition of inequitable (undemocratic) practices of school mathematics. Three such examples follow:

It's crucial, at least in my opinion, to present mathematical concepts within the proper context and to give material a meaning beyond mere textbook definitions and generic examples. It is our responsibility as educators to have a strong command of the material and to relate it to students in a way that is meaningful and purposeful to them. (Mr. Ahmed)

This course made me realize how unbelievably important it is to recognize that each student carries with him a personal set of experiences. Students of low socioeconomic status and of color are at a higher risk of experiencing events that diminish their self-worth. These students may have been told by past teachers that they are not capable of doing math. These students may have not had the resources or help at home to succeed in math. This has reminded me that not only do I have to provide equal opportunity and access to math within my classroom, but I have to make a conscious effort to include and support students that face additional struggles. (Ms. Ramirez)

Building upon what I have learned and experienced, I plan to integrate purposeful and genuine modes of dialogue between my future students and me. This type of dialogic relationship between student and teacher will foster the participatory learning process that students need in order develop a deeper understanding of mathematics in addition to helping me grow and develop into a better teacher. (Mr. Cortez)

These excerpts, both those focused on PSTs own knowledge of mathematics and those showing a concern for creating learning environments that are more participatory, are suggestive of a move toward more democratic practice in the teaching of mathematics. The fact that these PSTs have developed such thoughts while in the process of forming professional identities as teachers of mathematics offers some hope of a greater awareness of how their classroom practices may or may not lead to more equitable outcomes for all students.

Conclusion

Given the continued reliance on traditional and less-than-democratic methods for teaching mathematics, it is imperative that a methods course for pre-service teachers of mathematics offer PSTs alternative experiences learning about mathematics as well as learning about teaching mathematics. The model developed herein based on principles of democratic education has helped me to better focus course assignments, readings, and activities on actively challenging PSTs to not just consider different approaches to teaching mathematics but to reconceptualize their image of what is possible in mathematics teaching.

In implementing the strategies described above, there have been challenges due in large part to what mathematics educator Renuka Vithal (1999) has termed the *complimentarity* between democracy and authority. As program advisor and instructor, my position relative to the pre-service teachers carries greater authority. In turn, pre-service teachers entering the methods course are often surprised at being asked to take on roles typically associated with the teacher—explaining problems, leading discussions, and evaluating the performance of others. The apprehension and occasional tension this generates are taken as indicators that a move toward a more democratic community is underway. As this work is ever an on-going process, these practices will continue to be refined in response to reflection on my students' needs, preferences, and reactions. In turn, it is hoped that through modeling in the methods course more democratic interactions among teacher, students, and knowledge, these PSTs envision possibilities to do the same in their own mathematics classrooms.

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Appendix 1: Class Discussion Facilitation Rubric

Task: Two students will be assigned one reading for which they will facilitate a class discussion. Before leading the class discussion, they must develop a set of questions and prompts and review these with the instructor. One the day of the discussion, they will bring copies of the questions and prompts for each member of the class and conduct a 20-minute discussion with the class. The rubric below should be used for planning and will be completed by each participant after the discussion.

Dimension	Qualities of Exemplary Work	Pts	Comments
Preparation	• Provided copies of questions and prompts to classmates • Handout makes direct reference to the assigned reading • Clear evidence throughout the discussion that the article was read thoroughly ahead of time	/5	
Discussion Questions	• Relevant to advanced knowledge of methods for teaching mathematics • Clear and understandable • Referred to reading • Encouraged students to refer to reading • Drew out and built on main points • Extended our thinking to make important connections	/5	
Discussion Methods	• Teaching methods o Engaged students- motivating o Introduced clearly o All voices heard • Guided but did not dominate discussion • Summarized main points or perspectives • Discussion with different viewpoints promoted, not a presentation	/5	
Facilitation Skills	• Made good eye contact • Demonstrated active listening • Utilized paraphrasing • Redirected questions to maintain focus • Kept the discussion positive and productive	/5	

Formative Feedback for Individuals

Name	Name

5

Engaging Preservice Teachers in Textbook Analyses: A Focus on Mathematical Tasks

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One of the central components of pedagogical content knowledge (PCK) is curricular knowledge (Grossman, 1990). Grossman defines curricular knowledge to include “knowledge of curricular materials available for teaching particular subject matter, as well as knowledge about both the horizontal and vertical curricula for a subject” (p. 8). Given the current availability of different types of mathematic textbooks, i.e., “problems-based” vs. “traditional” textbooks, I contend that curricular knowledge for mathematics teachers also includes being able to articulate differences in these types curricular materials.

In previous work (Arbaugh & Brown, 2004; Arbaugh & Brown, 2005), I have argued that learning to analyze mathematical tasks using the Levels of Cognitive Demand framework (Stein, Smith, Henningsen, & Silver, 2000) influences the ways that teachers view mathematical tasks as well as their choice of tasks to use with students. From my work as a university mathematics teacher educator, I have found the Levels of Cognitive Demand framework to be equally powerful when engaging pre-service teachers in textbook analyses. In this manuscript, I build on my previous work by sharing two textbook analysis assignments that I have used with pre-service teachers (PSTs) to focus them on the mathematical tasks contained in mathematics textbooks, and thus be able to differentiate between the types of textbooks based on the levels of thinking required by the mathematical tasks contained in those textbooks. Although my work has been with secondary PSTs, the ideas presented here would work equally well with elementary PSTs.

Prior to the Textbook Analysis Assignment

In order for my PSTs to be able to complete a textbook analysis assignment, they need to learn about the Levels of Cognitive Demand framework. I engage them in a task-sorting activity designed specifically to support their learning of the Levels of Cognitive Demand framework (see Stein, Smith, Arbaugh, Brown, & Mossgrove [2004] for the task-sorting activity and sets of mathematical tasks at the elementary, middle, and high school levels). We complete this activity relatively early in the semester, as it impacts course content beyond the textbook analysis assignment. For readers unfamiliar with the Levels of Cognitive Demand framework, I strongly suggest reading *Implementing Standards-Based Mathematics Instruction: A Casebook for Professional Development* (Stein, Smith, Henningsen, & Silver, 2000) as well as the chapter written by myself and Catherine Brown in *Perspectives on the Teaching of Mathematics* (Arbaugh & Brown, 2004), as both of these resources provide important background for implementing the task-sorting activity.

The Textbook Analysis Assignments

In this section, I present two different textbook analysis assignments I have used with my PSTs that focus them on examining mathematical tasks through the lens of the Levels of Cognitive Demand framework. The assignments are slightly different, for reasons that I will explain. In presenting the assignments here, I reference the specific mathematics textbooks that I use for the assignments. I do so to provide some context for the assignments. In the end, it is not important that you use the exact same textbooks as I use; I will provide characteristics of the textbooks I think are important so that other mathematics teacher educators can use textbooks that they have access to and are relevant to their contexts.

Textbook Analysis Assignment 1: Concepts of Algebra

One of the textbook analysis assignments that I use with my PSTs focuses them on the content area of algebra. I organize this work into two phases. In Phase 1, students complete a take-home textbook analysis assignment. Phase 2 consists of an in-class analysis and accompanying whole-class discussion.

Phase 1. Each student receives a student copy of *Moving Straight Ahead*, a 7th-grade algebra unit from the Connected Mathematics Program (CMP) (Lappan, Fey, Fitzgerald, Friel, & Phillips, 2002). I use this particular unit and textbook series for a number of reasons: 1) the content of linear relationships maps nicely onto the content of the linear equations chapter of a typical Algebra I (or Algebra II) textbook; 2) this program is used in middle schools in the local district; 3) it is representative of a “problems-based” mathematics textbook, where students are expected to learn mathematics by engaging in collaborative problem solving; 4) it contains a wide variety of mathematical tasks that can be categorized through the Levels of Cognitive Demand framework; and 5) practically, I have access to multiple copies of the unit to lend to students.

Note that the Phase 1 assignment has two different parts (see Figure 1), typically due about three weeks apart. First, I require that my PSTs actually engage in doing all of the tasks in two complete sections (in this textbook series, sections are called "investigations" of the unit). This work affords them the opportunity to develop a student's perspective of the type of thinking required by the tasks.

Textbook Analysis Assignment

Part 1; Due Thursday, February XX

Individually, do ALL of the tasks in the first two investigations of your unit (including Problems and Follow-Ups, Applications, Connections, Extensions, and Mathematical Reflections).

You will hand in your work on these problems. Organize it in any way that makes sense to you; I need to be able to understand what you have done and find specific problems that I want to look at.

Part 2; Due Thursday, March XX

As a pair, write a "report" on your unit. Be sure to include discussion regarding the following issues:

1. Categorize each task in the unit (not just the ones you worked on) with regard to its required level of cognitive demand. Report the number and percentages for each level of cognitive demand. Give two examples of tasks that require a high level of cognitive demand and explain why you categorized them as such. Give two examples of tasks that require a low-level of cognitive demand and explain why you categorized them as such.
2. What types of assessment are included in this unit? Do these assessment tools mostly assess procedural or conceptual knowledge? Are the assessments formative or summative in nature?
3. Describe the use(s) of technology in this unit. How does the use of technology compare to the recommendations made in *Principles and Standards for School Mathematics*?
4. Include any other descriptions of the unit that could help the rest of us make an informed decision about its use in learning/teaching algebraic concepts.

As individuals, write a 1- to 2-page reflection about doing the problems. Write this reflection from two viewpoints: that of a learner and that of a future teacher. Attach these reflections onto your report.

Figure 1: Algebra Textbook Assignment

In addition to focusing the PSTs on the levels of cognitive demand required by the mathematical tasks contained in this CMP unit, I also ask that they attend to the assessments that are built into the unit as well as how students use technology in the unit. Often I find that my PSTs have difficulty recognizing the opportunities for formative assessment that are built into the student edition of a CMP unit (e.g., mathematical reflections assignments; the summary that occurs at the end of each problem in an investigation). Focusing the PSTs on the use of graphing calculators, particularly in this unit, allows them to see curricular materials written so that students use the graphing calculator to learn mathematics, not as a "crutch."

Using these guidelines, the PSTs complete Phase 1 in pairs and submit a final report of their work. I then grade this part of the activity and we move into Phase 2.

Phase 2. During the class period when I hand back the PSTs' graded CMP analysis assignment, we engage in a second textbook analysis. First, using a pre-constructed chart on the white board (see an example in Figure 2; this could also be done on an overhead projector), students record the data from their CMP analysis assignment. During the whole-class discussion of the compiled data, I make sure that the following points are made:

1. The percentage of tasks categorized as "procedures without connections" is similar to the percentage of tasks categorized as "procedures with connections." This comparison is important for the PSTs to consider, given the claims, often from uninformed detractors, that CMP pays little attention to the skills of mathematics. It is valuable for the PSTs to see that the tasks they categorize as "procedures without connections" are those tasks included in the curriculum materials that require students to build and practice mathematical skills.

	% of tasks categorized as "Memorization"	% of tasks categorized as "Procedures without Connections"	% of tasks categorized as "Procedures with Connections"	% of tasks categorized as "Doing Mathematics"
Group 1				
Group 2				
Group 3				
Group 4				
…..				
Group n				

Figure 2. In-class Chart #1

2. Tasks that require different levels of cognitive demand are “scattered” throughout each unit. In other words, the teacher would have a very hard time assigning just the tasks that require one level of cognitive demand and ignoring others that require a different level of cognitive demand. This point becomes important as we move into the next part of this textbook analysis work.

Following our discussion of the compiled data from CMP, I engage the PSTs in analyzing the linear graphing chapter from a representative Algebra I textbook. I give each pair a Xeroxed copy of one section of the chapter (all of the tasks available for students to complete), and ask them to categorized the tasks using the Levels of Cognitive Demand criteria. Upon completing the analysis on their section, PSTs record the data on the white board in a chart similar to the one we completed on CMP (see Figure 3). Note that we now have two charts on the white board, one that contains the data from the CMP unit and one that contains the data from the Algebra I chapter.

	% of tasks categorized as “Memorization”	% of tasks categorized as “Procedures without Connections”	% of tasks categorized as “Procedures with Connections”	% of tasks categorized as “Doing Mathematics”
Section 1				
Section 2				
Section 3				
Section 4				
….				
Section n				

Figure 3. In-class chart 2

Comparing the data on the two charts always has quite an impact on the PSTs. Their analysis of the Algebra I chapter indicates that a large majority of tasks can be categorized as “procedures without connections” with much smaller percentages (typically 0-10%) falling into the other three categories. Subsequent whole-class discussion often includes the following:

1. The PSTs notice that while the Algebra I chapter does contain tasks that require a high level of cognitive demand, those tasks are most often found at the end of a section, where

it would be easy for a teacher to choose not to assign the problems. I ask them to reflect on their own experiences in Algebra I and very few report being assigned those "word problems" at the end of each section.

2. PSTs are able to understand why they are so good at performing procedures and are less confident in their conceptual understanding of high school algebra. The large majority of my PSTs report learning algebra from a textbook like the one we use for this activity. Once they have analyzed the tasks through the Levels of Cognitive Demand lens, they see that they were given many opportunities to learn "procedures without connections" and far fewer opportunities to learn "procedures with connections" or engage in "doing mathematics." In some ways, this comforts the PSTs, who now understand *why* they do not understand mathematics in the way that I have been asking them to know it in this class – they know very well the mathematics they were given the opportunity to learn. In other ways, this experience makes them nervous about teaching high school mathematics, which often prompts my PSTs to design final projects through which they "relearn" some aspect of high school mathematics.

Implemented as a full activity, the different phases of this textbook analysis support my students' knowledge of different types of curriculum materials. I use it with my undergraduate PSTs in a traditional teacher education program. In the next section, I share a second textbook analysis activity I designed for use in our post-baccalaureate certification program.

Textbook Analysis Assignment 2: Comparing Geometry Textbooks

I designed this textbook analysis assignment for use in a secondary mathematics methods course that is a component of our post-baccalaureate certification program. PSTs in this program are considered "non-traditional students" – they are adults who have "come back" to college to get a teaching degree after previously obtaining a baccalaureate in a non-teaching area. Consequently, we believe that they need to spend a lot of time in a classroom as a part of their program. Full-time students in this program are placed in a mathematics classroom for 20 hours per week for two semesters; they work closely with the mentoring teacher, who has primary responsibility for the classroom in which the PSTs are placed. Other students in this program are full-time teachers, and thus are responsible for their own high school classrooms. Their program of study includes mathematics methods courses that meet on 5-6 Saturdays during the academic year (to accommodate the full-time teachers as well as students who live across the state).

Given these programmatic circumstances, I designed this textbook analysis assignment to utilize a textbook from their home school and to introduce them to *Connected Geometry* (EDC, 2000), an investigative- and problems-based Geometry textbook. I use this textbook for the same reasons I use CMP in the assignment I described above – it requires students to learn geometry concepts through engaging in problem-solving, and our mathematics education group has a full set of these textbooks that I can lend out to students.

The textbook assignment (see Figure 4) requires the students to identify a particular set of content in their school's Geometry textbook (in this case it is "circles," but could be any content that will map onto the *Connected Geometry* text), analyze the mathematical tasks within the sections they identified through the Levels of Cognitive Demand framework, and then describe the nature of the mathematical tasks. I then ask them to find the same content in the *Connected Geometry* text and perform the same analysis and description.

Textbook Analysis Assignment

This assignment will be completed with a partner. Please complete the assignment in the order of activities listed below.

1. Using a Geometry textbook from one of your schools, identify where students learn about circles. Analyze, using the **Levels of Cognitive Demand** criteria, the mathematical tasks that are available for students to complete. Create a table that has section number on the vertical and the following headings on the horizontal:

- Number of tasks in each section available for students to complete
- Number of tasks that require a high level of cognitive demand
- Percentage of tasks that require a high level of cognitive demand
- Number of tasks that require a low level of cognitive demand
- Percentage of tasks that require a low level of cognitive demand

2. Based on this analysis, describe the nature of the mathematical tasks found in your textbook. Include a discussion regarding the placement of tasks with different levels of cognitive demand within each section. Use specific tasks as examples to support your description.

3. Map the content of each section onto the *Connected Geometry* text. Create a table that displays the mapping. Be specific (Map your text's sections onto *Connected Geometry* chapter/section/page numbers).

4. Using the specifics from #3, analyze, using the **Levels of Cognitive Demand** criteria, the mathematical tasks you identified in *Connected Geometry* that are available for students to complete. Create a table similar to the table you created for #1.

5. Based on the tasks contained in your mapping from #3, describe the nature of the mathematical tasks found in *Connected Geometry*. Include a discussion regarding the placement of tasks that require different levels of cognitive demand within each section. Use specific tasks as examples to support your description.

6. John Van de Walle, the author of the most-used elementary mathematics methods book in the U. S., uses the following list of words to describe the "verbs of mathematics":

Explore	Investigate	Conjecture	Solve
Construct	Discover	Represent	Formulate
Communicate	Justify	Verify	Explain
Predict	Develop	Describe	Use

For each textbook, use these words to write a paragraph that characterizes the nature of the work that students would do if they used the text. Provide examples from each text to illustrate your characterizations.

7. For each textbook, write a paragraph that characterizes the nature of the teacher's role in a classroom where the text is being used.

8. In summary, reflect on the similarities and differences between these two geometry textbooks. Include other reflections about what you learned through this textbook analysis assignment.

Figure 4: Textbook Analysis Assignment 2

This textbook analysis assignment differs from the first one in a few important ways. First, through the use of Van de Walle's (1994) "verbs of mathematics," I focus these PSTs on the mathematical activity high school students will "do" when engaging in the mathematical tasks from each textbook. Through this lens, the PSTs realize that students who use a textbook like *Connected Geometry* are "doing" the type of mathematical activity that mathematicians "do." Second, I ask the PSTs to think about the role of the teacher (which could be an adaptation for the first assignment).

Summary

Each semester, in an effort to get to know my PSTs, I ask them to describe the book that they used in algebra (or geometry) class in high school. A large majority of the PSTs describe what we would consider to be a "traditional" textbook, with each section starting with examples of how to solve the problems in the section followed by a lot of problems to solve with some word problems at the end of the section. The textbook analysis assignments I describe in this article expose my PSTs to a different kind of mathematics textbook than the ones they used to learn mathematics in high school. At the same time, these assignments equip them with a framework through which to critically analyze the content of different textbooks – a framework that focuses on student thinking and their opportunities to learn.

My PSTs and I revisit the results of these textbook analyses activities throughout the semester in many different ways. For example, PSTs search the web, seeking classroom activities containing mathematical tasks that require a high level of cognitive demand that they could use to supplement a traditional text. They examine the tasks used in their field classrooms through the lens of the Levels of Cognitive Demand framework. Ultimately, the textbook analysis activities, I hope, will support them in the future as they make the day-to-day decisions about tasks to use with their mathematics students.

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6

Including English Learners in Secondary Mathematics Methods Courses

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Abstract

Because the requisite pre-service training to meet the mathematics learning and performance needs of secondary English learners often lies between a secondary mathematics methods course and a literacy/English language development course, secondary mathematics teacher candidates often must integrate the content of these two courses on their own, with varying levels of success. Based on my experience interweaving mathematics and language learning, teaching, and assessment into my practice, this paper integrates theory, research, and state/national standards to delineate secondary mathematics methods course content to help secondary mathematics teacher educators empower secondary mathematics teacher candidates to meet the mathematics learning and performance needs of secondary English learners. Course content, rationale, activities, and development frameworks are shared.

Acronyms in the article

CELDT	California English Language Development Test	**NCLB**	No Child Left Behind
CMC	Conservation of the Mathematical Construct	**PACT**	Performance Assessment for California Teachers
CTs	Cooperating Teachers	**SELs**	Secondary English learners
ELs	English Learners	**SMTs**	Secondary Mathematics Teachers
ELD	English Language Development	**SMTCs**	Secondary Mathematics Teacher Candidates
ELP	English Language Proficiency	**SMTEs**	Secondary Mathematics Teacher Educators
MKT	Mathematics Knowledge for Teaching	**TPEs**	Teaching Performance Expectations

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Across the country, English learners[1] (ELs) are receiving greater attention from mathematics educators than ever before, in part because of their growing numbers - 5.5 million in the U.S. (U.S. Department of Education [USDOE], 2004) - and their explicit inclusion in current state accountability systems under No Child Left Behind (NCLB) (2002). In California, where there are more than 600,000 ELs in grades 6 – 12 (California Department of Education [CDE], 2008), our *Mathematics Framework* (Curriculum Development and Supplemental Materials Commission—Mathematics [CDSMCM], 2006) expects secondary mathematics teachers to pay attention to the learning and assessment needs of ELs (p.339). Appendix A of our *Standards of Quality and Effectiveness for Teacher Preparation Programs for Preliminary Multiple and Single Subject Teaching Credentials* (California Commission on Teacher Credentialing [CCTC], 2007) explicates Teaching Performance Expectations (TPEs) that California pre-service secondary mathematics teachers are expected to meet for teaching their ELs. Nationally, the *Standards for the Education and Continued Professional Growth of Teachers of Mathematics* (National Council of Teachers of Mathematics [NCTM], 2007) provides a general framework for secondary mathematics teacher educators (SMTEs) to use to help secondary mathematics teacher candidates (SMTCs) meet the TPEs.

However, we SMTEs lack specific guidance and training from within our field to enable our secondary mathematics teaching candidates to meet fully these expectations. Though we have support from our literacy and bilingual education colleagues within university teacher education programs, because they usually do not possess the requisite mathematics content knowledge, mathematics knowledge for teaching (MKT) (Ball & Bass, 2003) and mathematical pedagogical content knowledge (PCK) (Shulman, 1986; Hill, Ball, & Schilling, 2008) that we do, we should be leading the way. However, because we typically do not possess the English language development (ELD) content knowledge, knowledge for teaching, or PCK they do, we need assistance. To help foster dialogue and collaboration with our literacy and bilingual education colleagues, model such school site collaboration for our secondary mathematics teacher candidates (Nordmeyer, 2008), and provide direction for ourselves, the following question is asked:

What should be in a secondary mathematics methods course to help SMTEs empower SMTCs to meet the mathematics learning and performance needs of SELs?

Addressing this overarching question requires the consideration of many prerequisite and related questions. Within and across the SEL, the SMTC, and the SMTE levels, these related questions are grouped under three broad sub-questions.

1) What should SMTCs learn to do to begin meeting the needs of their future SELs?
2) How and to what extent can SMTEs facilitate such SMTC learning?
3) What frameworks can SMTEs use to create and evaluate effective activities for SMTs that address SEL needs?

Answering sub-questions 1–3 helps we SMTEs improve our SEL-related PCK to initiate SMTC transformative learning, a long-term goal and process when SMTCs raze their long-standing beliefs, reorganize their understandings, and restructure their core principles for learning (Mezirow, 1991, 1997; Thompson & Zeuli, 1999).

What Should SMTCs Learn to Do to Begin Meeting the Needs of Their Future SELs?

Contrasting statements made by the 3rd National Council of Teachers of Mathematics (NCTM) president in the first NCTM yearbook (Schorling, 1926) and the 41st NCTM president about the *Principles and Standards for School Mathematics* (Stiff, 2000; NCTM, 2000) shows the nation's mathematics teaching leadership has moved away from an EL-deficit framework toward one of accommodation and inclusion**.** Following NCTM's lead, SMTCs should learn to design and actualize daily lesson plans that explicitly and simultaneously develop mathematics meaning-making and communication. Candidate-designed formative and summative mathematics assessments should be language sensitive and context embedded, but purposefully constructed to minimize unwanted ambiguity and confusion. To actualize these objectives, SMTCs must first get to know their SELs' abilities, ideas, and interests (TPE 8) as well as their mathematics and language proficiencies (TPE 7).

More generally, SMTCs should also know key information about state and U.S. EL communities at large. Though over 85% of California's ELs speak Spanish as their primary language, the remaining 15% speak one of at least 55 other languages (CDE, 2008). In addition, many ELs are U.S citizens, U.S-born children of immigrants, not immigrants themselves (Tafoya, 2002). Though SELs are heterogeneous across many characteristics, including mathematics proficiency, English language proficiency (ELP), primary language proficiency, and socioeconomic status (Gitomer, Andal, and Davison, 2005; Meltzer & Hamann, 2004), most are homogeneous in their desire to learn and willingness to invest the time and energy necessary to do so in a supportive environment (Meltzer & Hamann, 2004).

Second, SMTCs should know SELs' mathematics learning and performance needs. SELs need to learn and be assessed on grade level mathematics content. Because most secondary mathematics instruction and assessment is done exclusively in English, SELs need their content knowledge and ELP[2] for mathematics learning and assessment to be simultaneously developed. Though a strong California English Language Development Test (CELDT)[3] score can indicate advanced ELP generally, it does not mean the student has gained sufficient academic language competency for grade level mathematics learning and assessment.

TPE 4–Making Content Accessible–states, in part, that candidates must help their students develop their abilities to use and understanding academic language as well as teach them strategies to read and comprehend a variety of mathematics texts and information sources (CCTC, 2007). Though the academic language of mathematics is a construct without a shared, precise definition, the *California Mathematics Framework* (CDSMCM, 2006) expects all mathematics teachers to help ELs understand the academic language of mathematics instruction and assessment, with special attention paid to mathematical vocabulary (p. 339). Nonetheless, the field generally agrees mathematical academic language is comprised of several competencies beyond mathematics vocabulary knowledge, including mathematical language functions and structures (Coggins, Kravin, Coates, & Carroll, 2007), such as generalization, induction, and justification (California Analysis Team [CAT], 2008).

Through a systemic functional linguistics perspective, part of our academic language resides in the mathematics register, defined by Halliday (1974) as:

> …a set of meanings that is appropriate to a particular function of language, together with the words and structures which express these meanings. We can refer to a "mathematics register," in the sense of the meanings that belong to the language of mathematics (the mathematical use of natural language, that is, not mathematics itself), and that a language must express if it is used for mathematical purposes. (p. 68)

Spanos, Rhodes, Dale, and Crandall (1988) explicated some of the syntactic, semantic, and pragmatic aspects of this register[4]. Other researchers (e.g., Cuevas , 1984; Lager, 2006) have also added to and refined the register, while Schleppegrell (2007), in a synthesis of the linguistic challenges of mathematics learning and teaching, organizes several classroom aspects of the mathematics register into two main categories, multiple semiotic systems (symbolic notation, oral language, written language, graphs and visual displays) and grammatical patterns (technical vocabulary, dense noun phrases, being and having verbs).

Fillmore and Snow (2005) provide specific examples of academic language functions for high school exit exams that apply to secondary mathematics:

1) Extract meaning from texts and relate it to other ideas and information.
2) Extract precise information from a written text and devise an appropriate strategy for solving a problem based on information provided in the text.
3) Recognize and analyze textual conventions to trigger background knowledge.
4) Interpret word problems – recognizing that in such texts, ordinary words may have specialized meanings.

Because a SEL's ELP influences performance on content-area assessments administered in English (García & Menken, 2006; Menken, 2008), these academic language needs are likely reflected in the sizable, persistent state and national mathematics achievement gaps on large-scale mathematics assessments for SELs compared to secondary non-ELs (CDE, 2007b); National Assessment of Education Progress [NAEP], 2008). For example, between 2003 and 2007, the California High School Exit Examination—Mathematics (CAHSEE-M) pass rate for ELs was only 48% (n = 320,285), compared to over 80% for non-ELs (n = 1,540,417) (CDE, 2007b).

In addition to academic language needs in English, SELs often bring valid, non U.S-standard algorithms and problem solving approaches that need to be recognized by the SMTC. Also, some of the newest SELs need to become acquainted with U.S. mathematics classroom norms, like when calculator usage and collaborative work modalities are encouraged, not punished. Finally, SELs benefit from a welcoming, positive classroom environment where mathematics and language learning happens, not derision for mispronunciations or accents (TPEs 1b and 11).

Though Asturias, Dorph, Goldstein, and Lee (2006) concluded that there is a dearth of relevant, rigorous research documenting effective instructional strategies to meet the aforementioned EL learning needs, they identify eight Promising Practices that can promote EL understanding through student-student and student-teacher discourse:

1) Promoting communication about mathematics.
2) Incorporating writing into mathematics instruction.

3) Providing students opportunities to work in groups.
4) Supporting students to engage in metacognitive learning strategies.
5) Allowing students to work in their primary language.
6) Providing students with multiple representations of mathematical concepts and relationships.
7) Supplementing textbook materials with supplementary materials specifically designed to support ELs.
8) Engaging and building upon students' prior knowledge.

Meltzer and Hamann's (2004; 2005) syntheses of best practices for developing secondary EL literacy across content areas, including mathematics, echo many of the aforementioned practices but add four more Promising Practices:

9) Paying specific attention to improving reading comprehension through teacher modeling, explicit strategy instruction in context, and use of formative assessment.
10) Connecting content to students' lives.
11) Creating responsive classrooms.
12) Promoting student interaction with each other and with text.

In terms of assessment evaluation, creation, and revision, the Conservation of the Mathematical Construct (CMC) work (Lager & Petit, 2005; Petit & Lager, 2003) provides guidelines for iteratively fine-tuning the content, cognitive demand, context, language, and format of an assessment item to maximize its accessibility, without altering the mathematical construct being assessed. Building off of previous related work (Abedi & Lord, 2001b; Kopriva, 2000; Hanson, Hayes, Schriver, Le Mahieu, & Brown, 1998) the CMC guidelines for preparing tasks to assess SELs are:

1) Explicitly align the item/task with the targeted content and ELD standards.
2) Determine the desired level of cognitive demand.
3) Embed the item in a rich context (when appropriate).
4) Streamline the language
5) Appropriately use graphics, pictures, graphs, tables, diagrams, and models when they are appropriate.

In sum, SMTCs should know general SEL learning and assessment needs, be able to identify basic learning and assessment needs of their future SELs, and use Promising Practices and the CMC to address those needs.

How and to What Extent Can SMTEs Facilitate Such SMTC Learning?

To begin fleshing out some of the Promising Practices as well as the CMC, I report activities that I have created or co-created because they seem to meet SMTC need and because I am most familiar with their facilitation. Though not exhaustive, the EL Experience, Standards, Assessment, and Reading Comprehension activities provide examples of what can be done, how to do so, and why. Fewer details are shared for an activity when articles and/or websites exist to disseminate supporting materials and further information. Each activity is explicitly linked to the aforementioned TPEs, SMTE Standards (NCTM, 2007), and Promising Practices.

EL Experience

Because many SMTCs grow up fluent English speakers, they often have enjoyed mathematics instruction and assessment exclusively in English and have had very few, if any, mathematical learning and teaching experiences in a language other than English. As a result, they often enter a secondary methods course with many pre-conceived, and often ill-informed, notions about who ELs are and how and what they should be taught. Believing many ELs to be illegal immigrants and that because mathematics concepts are universal, mathematics is "language-free," are two such examples. Therefore, before discussing any EL-related standards, TPEs, academic language, or teaching strategies, I first facilitate The Travieso Activity (Lager, 2008a). This 90-minute workshop session enables SMTCs, as learners, to briefly experience meaning making and communication situations that SELs typically experience in English-only mathematics classrooms.

Asked to solve one mathematics problem in a Spanish-only environment, SMTCs systematically engage with the problem under several different problem presentation/problem collaboration stages to model their (in) effectiveness. For example, to model the ineffectiveness of talking at kids and socially isolating them, I first read the problem out loud in Spanish without any accompanying writing, visuals, or manipulatives and force SMTCs to work silently and independently. After each stage, SMTCs write down what they know about the problem, what would help them solve it, how they're feeling, and their confidence in their current answer.

After the last stage, I lead a whole-class debriefing so SMTCs can discuss the experience and share lessons learned.

During the debriefing and on their exit slips[6], SMTCs routinely report that they had underestimated and/or not anticipated the frustrations and linguistic challenges they experienced and wanted to know how to help SELs negotiate them. Their newfound awareness of how their own EL values and biases can affect EL learning as well as their own teaching practices meets TPE 12 and Standard 3; their desire to improve their developing practice for SELs as a result of doing and reflecting about the activity meets TPE 13 and Standard 5. The experience also stimulates a genuine need for Promising Practices 1, 3, 5, and 12. In addition, the experienced cognitive dissonance is a critical prerequisite for transformative learning.

Spanish is purposely chosen as the language of the activity because the range of participants' Spanish proficiency parallels the range of ELs' English proficiency. To see a similar simulation that forces almost all participants to experience doing mathematics like a beginning SEL, see Anhalt, Ondrus, & Horak's (2007) Chinese activity. For more on this activity, read "The Travieso Activity" (Lager, 2008a) and visit www.Traviesoactivity.com.

Travieso follow-up.

At the conclusion of the activity, I assign related readings to build upon the Travieso experience. For example, "Teaching English-Language Learners (ELLs) 'The Language Game of Math'" (Gebhard, Hafner, & Wright, 2004) provides a brief case study of a Spanish-speaking ELL's mathematics learning and performance challenges and growth over one year's time. "Adapting Mathematics Instruction for English-Language Learners: The Language–Concept Connection" (Garrison & Mora, 1999) provides a four-domain heuristic to consider the relationship between mathematics content and language in the classroom. Moschkovich's (2007) "Bilingual Mathematics Learners: How Views of Language, Bilingual Learners, and Mathematical Communication Impact Instruction" compares three theoretical views of bilingual mathematics learners to argue that a sociocultural, discourse frame informs the field's understanding of how ELs concomitantly learn mathematics and English better than a systemic functional linguistic frame that focuses narrowly on vocabulary and multiple meanings (Halliday & Mattheisen, 2004; Schleppegrell, 2004).

Collectively, these three readings lay out language-mathematics interactions and feelings for SMTCs to consider when teaching SELs, introduce teaching strategies to address those interactions and feelings, and compare/contrast theoretical frames through which to view SEL mathematics learning and performance needs and the teaching strategies to address them. To ensure SMTCs read the readings, apply their readings to their secondary mathematics classroom observations, and are prepared for the homework-based follow-up activities that take place in the next class session, specific guiding questions/directives are assigned. Because these questions will be revisited and expounded upon further throughout my course, in their Literacy course, and in doing their Performance Assessment for California Teachers (PACT)[7] work, I ask SMTCs to write only 1-2 paragraph responses for each:

1) Describe one interaction you have witnessed with SELs themselves or with the cooperating teacher (CT)[8]/paraeducator that helped SELs better understand the mathematics and language. Which of the Garrison and Mora domain(s) were each of the participating SELs before the interaction and after?
2) Describe one interaction you have witnessed with SELs themselves or with the CT/paraeducator that hindered SELs from understanding the mathematics and language. Which of the Garrison and Mora domain(s) were each of the participating SELs before the interaction and after?
3) Looking back at the interaction from question 2, what strategies could the CT/paraeducator have employed to change it from one that hindered to one that could have helped?
4) Of the three frameworks presented by Moschkovich (2007), through which framework(s) does your CT view SEL mathematics and language learning? Support your judgment with at least 2 specific classroom examples.
5) Of the three frameworks presented by Moschkovich (2007), through which framework(s) do you currently view SEL mathematics and language learning? Why?
6) Do a 10-minute interview with one SEL in your observation classroom. Ask him/her
   a) how he/she feels about learning mathematics in English
   b) what the challenges are
   c) what helps her learn
   d) what would help her learn better
   e) if she could change one thing about learning mathematics, what would he/she change?

Summarize your interviewee's feelings and insights.

SMTCs email me their responses at least 24 hours before the next class so I can familiarize myself with their ideas ahead of time and identify patterns of convergence and divergence in their answers to explore in the first half of the next class. For example, if most of my SMTCs write that they view SEL mathematics and language learning through all three Moschkovich frameworks, I'd have each of them create a graphic organizer (on 8.5" x 11" paper), in class, to show how they view the relationships among the three. I would then have them share and listen to each others' ideas, in small groups of two to three persons first, and by small groups to the entire class second, to help them expand and hone their own conceptions.

Next, I pass out two handouts of all the interactions they documented from their classroom observations. To create these handouts, I copy and paste all the emailed responses to question/directive 1 into one document and responses to question/directive 2 into another, both without SMTC names. To keep anonymity across the responses, yet distinguish one CT from another, I rename the CTs, Mr. A, Ms. B, etc. I then give SMTCs time to read the handouts individually and silently one time through to familiarize themselves with the responses and a second time through to look for response patterns within each handout (e.g., perhaps some of the hindering interactions centered on non-reciprocal hearing between teacher and student). Next, I break SMTCs into groups of four to discuss what patterns they saw and how the patterns are related to each other, if at all. For example, maybe two of the documented helping interactions showed the teacher and student hearing each other well. Then, I facilitate a whole group discussion to share, discuss, and connect SMTC insights. During that discussion, I remind SMTCs that we are examining and critiquing interactions, not the CTs/paraeducators or the SELs themselves.

I then pass out a handout of all the interview findings and similarly proceed. A concluding whole group discussion synthesizes the work by 1) exploring connections between how CTs/paraeducators help/hinder EL mathematics learning and how ELs perceive such interactions and 2) interpreting these actions and perceptions through Garrison and Mora's (1999) and Moschkovich's (2007) frameworks to explore how and to what extent such interpretations support the argument for secondary EL mathematical discourse. To help facilitate such discussions, SMTCs are always required to bring hard copies of their homework responses to class.

The follow-up activity helps SMTCs check for and address common student misunderstandings (TPE 2), understand how cognitive, pedagogical, and individual factors affect students' language acquisition (TPE 7, Standard 3), learn about student abilities and ideas through interpersonal interactions (TPE 8), learn ways to promote student discourse (Standard 4) and communication about mathematics (Promising Practice 1), model a responsive classroom (Promising Practice 11), and recognize academic language in the mathematics classroom.

Pedagogical benefits.

There are many pedagogical benefits to this approach. In the aforementioned graphic organizer example, the lesson-generating process is not random. While SMTCs create their organizers, I walk around to see what they were constructing; while they dialogue in small groups, I sit down with each group to listen to the conversation. While silently observing both processes, I mentally note big ideas and tensions they were raising that I want to make sure everyone heard and discussed. In addition, any key points not raised by the groups, such as why many secondary mathematics textbooks focus almost exclusively on vocabulary acquisition for ELs to the exclusion of facilitating mathematics discourse for them, I would either hint at or explicitly put on the table for discussion. These collaborative norms, where my students and I learn from each other, are explicitly built and maintained in my course and rooted in adult learning practice and theory (e.g., Edmondson, 2008)

Building upon homework answers through planned class activities exemplifies and values this learning/teaching reciprocity and models ways SMTCs can use small group and whole group interactions in their future classrooms. The original homework responses and handouts, upon which SMTCs are encouraged to inscribe notes from the small group and large group discussions, become written records of SMTC thinking. The readings are purposefully picked to meaningfully combine practice, research, and theory. Though all three readings suggest teaching strategies to help meet EL needs and share samples of student work, one focuses on the yearlong learning trajectory of one EL, one presents a more practitioner-oriented framework for the interactions between mathematics content and language, and the third a more theoretical-oriented framework of EL mathematical learning. SMTCs can reference the homework, handouts, and readings for their PACT and M.Ed. work and the SMTE can use them to iteratively research and improve the course.

After introducing SMTCs to SEL mathematics and language needs, I explicitly interweave SEL needs and teaching and assessment strategies throughout the rest of the course's weekly thematic units to model how SMTCs are expected to integrate mathematics content and English language development within their lesson planning and instruction. Specific examples follow for standards, assessment, and reading comprehension.

Standards

When I spend one class session introducing SMTCs to the *California Mathematics Framework* (CDSMCM, 2006) and NCTM's *Principles and Standards of School Mathematics* (NCTM, 2000), I make sure to have them locate and consider explicit EL-related parts, such as pages 234 (Universal Access) and 339 (English Learners) (CDSMCM, 2006) and pages 12–13 (Equity Principle) (NCTM, 2000), and identify implicit parts that require special attention for ELs, such as expecting students to read problems carefully (NCTM, 2000, p. 54) and become more precise with their written mathematics communication (NCTM, 2000, p. 351). In addition, the *California English Language Development Standards* (English Language Development Committee [ELDC], 1999) are included because they must be clearly linked to the *California Mathematics Standards* (CDSMCM, 2006) under NCLB (2002) and to bring explicit attention to the ELD necessary to engage with mathematics content, problem solve, and communicate mathematical understanding.

Linking standards

To make these linkages visible, one homework activity I do is choose five mathematics problems from different grade levels (6 – 12), content areas, and curricular materials, and ask SMTCs to determine which mathematics (CA and NCTM) and ELD standards apply to each problem. Because I require them to send me their responses ahead of time by email, I cut and paste their responses to form one combined class response (with no names) for each mathematics problem. Then, in the following class, I break the SMTCs up into pairs, hand out one mathematics problem and its combined class response to each group (but different problem/response for each group), and ask each pair to vet the response. Excising an inappropriate standard from the response requires the group to write a one to two-sentence justification. After vetting the item, the pair is then asked to construct a graphic organizer (on a large piece of butcher paper or digital whiteboard) to show the interrelationships of the mathematics and ELD standards for their mathematics problem.

Then, a 4-corner presentation/learning strategy is used. Each pair goes to a different part of the classroom, hang up their graphic organizer, and decide who will provide "patron service" first. Then, all the first providers, the "1"s, stand by their work while all the "2"s, the patrons, peruse the different graphic organizers and ask providers follow-up questions about their work. After 15 minutes or so, the "1s" and "2s" switch so that each provider becomes a patron and vice-versa. Afterward, I lead a whole-group discussion to explore the strengths and challenges of constructing these linkages, working in pairs, and doing the provider/patron strategy. For homework, I ask each SMTC to write a two-page paper explaining how they would apply the strengths and challenges discussed in class to lesson planning and teaching the mathematics and language involved in the one problem for which they were providers. After sharing and discussing their applications with each other during the next class session, SMTCs can incorporate them into their future lesson planning.

This activity is an important early step to helping SMTCs teach to state-adopted mathematics and ELD standards (TPEs 1b and 7), consider lesson planning for students' linguistic backgrounds (TPE 9), and connect state standards (when possible) to national mathematics content recommendations (Standard 2). In addition, the exercise provides an official rationale for using the Promising Practices. In the near future, an online tool will be available to help teams of SMTCs identify the academic language demands of the *California Mathematics Standards* for SELs (George Washington University Center for Equity & Excellence in Education [GW-CEEE], 2009).

Assessment

***Huerto de Manzanas*.**

Similar to how "The Travieso Activity" simulates common secondary EL mathematics classroom experiences and feelings, the "Huerto de Manzanas" activity (Lager, in press) models typical secondary EL large-scale and classroom mathematics assessment experiences and feelings. SMTCs silently and independently engage with four related, but modified versions of one released algebra item from the California High School Exit Examination (CDE, 2007c). Versions 1–3 are in Spanish and Version 4 is the actual item, in English. For each version, 90 seconds is given to read the item, solve it, and write down the answer. At the conclusion of each 90-second interval, participants are also asked to immediately write down their level of confidence in their answer (1-5), with 1 meaning not confident and 5 meaning confident.

Immediately afterward, SMTCs freewrite, then discuss in small group and whole group settings, the meaning-making and problem solving strategies they employed and the feelings they experienced during the simulation. Then they try to apply their newfound experience and knowledge to anticipate how secondary ELs might engage with the actual item.

During the discussions, SMTCs share employed problem solving strategies, such as looking for familiar words and symbols, like cognates, numbers, variables, and units, assuming that if information is given, it must be used, and working backwards from multiple choice answers. SMTCs also share their affective states, such as feeling frustrated for having difficulty accessing the Spanish mathematics problems, scared that they'll get the wrong answer, and uneasy about lacking confidence in their answers. Without realizing it, SMTCs often engage with the items in ways similar to their future EL students and temporarily share similar feelings as well. Afterward, SMTCs deconstruct the wording and linguistic structure of the item, in Spanish and English, pointing out its unintended and unnecessary language challenges. This activity helps them see the need for using multiple measures to gauge SEL mathematical understanding (TPEs 3 and 7; Standard 3) and for teaching SELs to use metacognitive learning strategies (Promising Practice 4). For more, visit www.Huertodemanzanas.com.

Conserving the Mathematics Construct activity.

After experiencing the "Huerto de Manzanas" activity, they want to know how to write and revise items that are more accessible for SELs. The "Conserving the Mathematics Construct Activity" can be used to help SMTCs consider the linguistic fairness of written mathematics items as well as keep SEL needs at the fore when creating new classroom mathematics assessment tasks. Because many mathematics assessment items do not take into account the needs of ELs when they are written, or do not do so sufficiently as seen in the Huerto de Manzanas activity, items often have to be modified to meet EL needs after they have been created for non-ELs. Unfortunately, these modifications, which include a visual and adding a context, often unintentionally introduce new challenges and alter the mathematical construct being assessed.

Before class, SMTCs read "Conserving the Mathematical Construct in item development while providing access to the greatest number of students" (Petit & Lager, 2003) and "Professional development: Universal Design and Conserving the Mathematical Construct" (Lager & Petit, 2005) to learn about its research- and practice-based genesis and its application to creating and improving elementary and middle school mathematics items. In class, I lead them in applying CMC to a high school mathematics item. For example, first they silently and individually solve a problem like this:

> A long pendulum hangs from the ceiling. As it swings back and forth, its distance from the wall varies sinusoidally with time. At time $x = 1$ s, it is at its closest point, $y = 50$ cm. Three seconds later it is at its farthest point, $y = 160$ cm. Sketch the graph. (Key Curriculum Press, 2007, p. 153)

Next, they work in groups of three to compare answers, discuss their solutions, interpretations, and drawings, and identify anticipated SEL meaning-making and meaning-sharing strengths and challenges for the problem.

While they are working, I am floating from group to group, listening for strengths, such as the pendulum context and drawing of the graph, and challenges, like the item's superfluous language and the need to generate appropriate scales for the graph, that should come out in the immediately following whole group sharing. Afterward, I have them work in pairs to apply the CMC to attempt to iteratively improve the item to address those predicted challenges while keeping the strengths intact and maintaining the integrity of the assessed mathematics construct. For example, in terms of streamlined language, to make clear what *it* and *its* refer to and eliminate *back and forth*, a participial phrase that is likely unfamiliar to an EL and unnecessary for this task, the item's second sentence might be changed to:

> *As the pendulum swings, the distance between the pendulum and the wall varies sinusoidally with time.*

Each pair then leaves their rewritten item on their table and walks around to see every other pair's revised item. A concluding whole group discussion compares different revisions, explores the rationales behind the revisions, and we arrive at the understanding that this work is messy and complex, but worthwhile. This activity helps them to learn when and how to use specialized assessments based on their ELs' linguistic and cultural needs (TPEs 3 and 7; Standard 3). For more, visit www.Huertodemanzanas.com and click on the "Conserving the Mathematical Construct" tab.

Reading Comprehension

Because many assessment items and textbook tasks do not follow CMC guidelines, helping SMTCs teach their SELs how to make meaning of tasks that are written in a less than ideal manner is important. The “Stamps and Change” and “Tennis Ball” activities model such reading comprehension strategies.

Stamps and Change activity

The “Stamps and Change” activity illuminates the value of and models the teaching of Hyde’s (2006) KWC and Checking Inference reading comprehension strategies to make meaning of and solve problems. These strategies are part of his overarching Braid Model of Problem Solving framework, which explicitly integrates reading comprehension with problem solving strategies. Though written for grade K–6 English speakers, his work applies directly to grade 6–12 ELs as well.

First, for this activity, SMTCs answer this problem’s three questions individually or in pairs:

> *A man buys 3-cent stamps and 6-cent stamps, 120 in all. He pays for them with a $5.00 bill and receives 75 cents in change. Does he receive the correct change? Would 76 cents change be correct? Would 74 cents change be correct? (Posamentier & Salkind, 1996, p. 13)*

Second, they go back and re-engage with the three questions, but this time by answering, in writing, five KWC and Checking Inference reading comprehension questions. Third, they compare their first set of answers to the second and justify why their answers did or did not change.

During the whole group debriefing that follows, the SMTCs and I unpack the disconnects between the implications communicated by the problem authors and the inferences made, often unconsciously, by themselves - the problem solvers. As a result, the SMTCs’ overarching algebraic metalinguistic awareness (MacGregor & Price, 1999) is raised, including the recognition of ambiguity (MacGregor & Price, 1999) and undesirable inferences (Wiest 2003), such as wordwalking (Mitchell, 2001) and confusing correlation with causation. Modeling the teaching and usage of Hyde’s strategies to help SELs read and solve algebraic word problems, as well as develop ELP teaches SMTCs how to foster improved content access and comprehension for SELs (TPEs 4, 7, and 9; Standards 3 and 4; Promising practices 4 and 9). For more, read “Reading Comprehension for Algebra Learning” (Lager, 2008b) and visit www.Huertodemanzanas.com, then click on the “Stamp and Change” tab.

Tennis ball activity

Though there are also many general reading comprehension strategies to help learners with reading across content areas (e.g., Vacca & Vacca, 2005; Strong, Perini, Silver, & Tuculescu, 2002), few are sufficiently explicated and modeled for secondary mathematics problem solving. The Tennis Ball activity (Lager & Laird, 2007) was created to model the combination and application of the following four general reading comprehension strategies to a typical secondary mathematics task:

1) Activate related prior knowledge and experience.
2) Break down the task into smaller chunks.
3) Use manipulatives or real-world objects.
4) Predict the problem.

Instead of attacking a task "cold" or being overwhelmed by text and not knowing where to start, this five-stage activity models one way a SMTC can actively unfold a typical task with SELs. At each stage, more task information than the previous stage is shared to help SMTCs predict the question/desired action. This unfolding/prediction cycle continues until the end of the fifth stage, when the task is given in its entirety. After doing the task, SMTCs discuss their answers, their progressions of their predicted tasks, and how and why their answers for the paper version of the task might differ from the real-world version. Though only one or two SMTCs will have predicted the actual task before Stage 5, most will have generated related tasks, "warming themselves up" by thinking about each piece of information and the relationships between them.

As they share and discuss their answers, they realize that one non-mathematical word and the differences between the paper/pencil and real-world versions of the task affect their answers and stimulate the kinds of conceptual understanding, strategic competence, and adaptive reasoning promoted by our field (National Research Council, 2001). Modeling the teaching and usage of general reading comprehension strategies to help SELs understand word problems and develop English language proficiency is an example of what SMTCs can do to help SELs foster improved content access and comprehension collaboratively as well as for themselves (TPEs 4, 7, and 9; Standards 3 and 4; Promising Practices 1, 2, 4, 8, 9, and 12).

For more on collaborative reading in mathematics, Barton and Heidema (2002) and the "Zoom In, Zoom Out" activities from Thompson, Kersaint, Richards, Hunsader, and Rubenstein

(2008) are good places to start. For more on this activity, visit www.Huertodemanzanas.com and click on the "Tennis Ball" tab.

More resources - Communication/Vocabulary/Literacy/Curriculum

Though space limitations do not allow other activities to be shared in this manuscript, for EL-related multimodal mathematical communication, see Morales, Khisty, and Chval (2003), Chval and Khisty (2001), Driscoll (2007; pp 100-101), and Brenner (1994). For more examples of classroom-based, EL mathematics teaching strategies that put Promising Practices into action, see Coggins et al., (2007), Fischer and Perez (2008), and Winsor (2007). For more information on mathematics reading (including vocabulary) and literacy development strategies, see Esty (2000), Barton and Heidema (2002), Murray (2004), Marzano and Pickering (2005), Thompson et al. (2008), and Vacaretu (2008). For an example of a state-adopted mathematics curriculum that explicitly considered SEL needs throughout and from its inception, see Garrison's (2007) LATCH model for the California adopted algebra readiness program *Intro to Algebra* (UCLA, 2007). For a comprehensive bibliography of linguistic and cultural diversity resources, join TODOS-Mathematics for All[9].

What Frameworks Can SMTEs Use to Create and Evaluate Effective Activities for SMTs That Address SEL Needs?

When iteratively creating, facilitating, evaluating, and revising the aforementioned activities, I found that the Loucks-Horsley, Love, Stiles, Mundry, and Hewson's (2003) professional development design framework, Levy's (2005) Effective Activity Cycle framework, and Guskey's (2000) evaluation model work well together.

Loucks-Horsley et al.'s (2003) professional development design framework provides a theoretical structure to construct a coherent set of SEL activities for a secondary methods course. The framework comprises six steps, Commit to vision & standards, Analyze student learning & other data, Set goals, Plan, Do, and Evaluate. Each step requires SMTEs to think on the SEL level and the SMTC level simultaneously. For example, at the SEL level, SMTCs should teach to the *California Mathematics Standards* (CDSMCM, 2006) and *California English Language Development Standards* (ELDC, 1999) and connect their practice to NCTM's *Principles and Standards of School Mathematics* (NCTM, 2000). At the SMTC level, we SMTEs teach to *California's Teaching Performance Expectations* (CCTC, 2007) and connect our work to NCTM's *Standards for the Education and Continued Professional Growth of Teachers of*

Mathematics (2007). Applying this standards duality to The Travieso Activity, for example, means that while the activity itself meets TPE 12 and Standard 3 at the SMTC level, the Travieso problem itself is aligned with California's and NCTM's algebra and geometry standards and connected to California's listening and reading comprehension ELD standards at the SEL level.

Levy's (2005) "Effective Activity Framework" is useful for developing activities that effectively challenge SMTCs' beliefs about teaching and learning mathematics, enhance their understanding of mathematics, or develop their PCK. Based on the research interviewing secondary mathematics methods instructors and examining their reform-oriented course syllabi, Levy (2005) found that effective pre-service activities must consist of three components:

1) Identification of an issue or dilemma
2) Data collection and reflection
3) Critical analysis and action

For example, because SMTCs know they will be observing and teaching SELs, a student population whose mathematics learning and performance needs they usually don't know well, some realize that to teach SELs effectively they need to better understand SEL needs. Once they identify this issue, they are ready for the data (e.g., personal beliefs and experiences, readings, student interviews, CT/paraeducator observations) and reflection (using the data to better understand SEL needs) generated from The Travieso Activity and its follow-up. Reevaluating their understanding of SEL needs and reconsidering their SEL beliefs are examples of resultant critical analyses and action for some SMTCs.

Guskey's (2000) evaluation framework provides a model for evaluating the efficacy of one activity or set of activities. Proceeding from the most rudimentary to the most complex, the model's five levels are: participants' reaction, participants' learning, organizational support and change, participants' use of new knowledge and skills, and student learning outcomes. Most activities' evaluations are typically limited to levels 1 and 2 due to the brevity of the methods course. For example, at level 1, SMTCs report that the SEL issues explored in The Travieso Activity and its follow-ups are relevant to their work and that the materials enhanced their learning; At level 2 they report that because they experienced a mathematics problem like a SEL and felt some SEL frustration, they take ownership of SEL needs and recognize the necessity of learning specific instruction and assessment strategies to address those needs. Looking at PACT work and M.Ed. projects at the end of the academic year or teacher induction work (like BTSA[10]) one to two years later could show evidence of success at levels 3 –5.

Conclusion

This work is a first step toward defining content and facilitation strategies for a secondary mathematics methods course to help SMTEs enable SMTCs to meet the mathematics learning and performance needs of SELs. This work should be used not only to help SELs and SMTCs meet content and teaching standards, but initiate a conversation of the related content knowledge and PCK SMTEs need to acquire to better assist SMTCs over the long term. Doing so will help us address documented SEL/non-SEL mathematics performance gaps by working toward eliminating the underlying opportunity gaps (Flores, 2008), when SELs have lesser access to qualified teachers, face lower expectations, and encounter less understandable assessment items than their secondary non-EL counterparts. SMTEs must work together with literacy and bilingual education colleagues to actualize these goals.

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Footnotes

[1] In California, English learners (ELs) are students for whom there is a report of a primary language other than English on the state-approved *Home Language Survey* **and** who, on the basis of the state approved oral language (grades K-12) assessment procedures and including literacy (grades 3-12 only), have been determined to lack the clearly defined English language skills of listening comprehension, speaking, reading, and writing necessary to succeed in the school's regular instructional programs (CDE, 2007a). In fact, because there is no definition of EL agreed upon by the United States Department of Education (USDOE) and the 50 state DOEs, subsets and supersets of ELs are also referred to in the research literature and field under many synonyms, including English-as-a-second language (ESL), English language learner (ELL), and limited English proficient (LEP). Though LEP is used by the USDOE, it is the least favored by the field because of its pejorative and misleadingly static connotation; because language learning is cumulative, an EL's English language proficiency is always improving.

[2] Listening, speaking, reading, and writing grade level English per California's English language arts standards

[3] California English Language Development Test, which is given annually to ELs to measure their progress in acquiring the skills of listening, speaking, reading, and writing in English.

[4] See pp. 25 and 26 from http://homepage.mac.com/gmuller/cmcblog/assets/author.pdf.

[5] English Only students (native English speakers) + Initially Fluent English Proficient students (students for whom English is not their home language, but upon first being tested on the CELDT demonstrated sufficient ELP not to be labeled an EL) + Redesignated Fluent Proficient students (former ELs who have attained sufficient ELP to no longer be labeled ELs) = non-ELs

[6] Courtesy of Elissa Ross, an exit slip is a 3-question evaluation sheet each SMTC fills out anonymously and turns in at the conclusion of each class. The questions are: What worked well today and why? What could've been improved and how? Any burning comments/questions to share?

[7] Performance Assessment for California Teachers (PACT) is a teacher performance assessment that an SMTC needs to successfully complete to earn a Single Subject Teaching Credential.

[8] A cooperating teacher (CT) is the credentialed mathematics teacher who supervises the SMTC when the SMTC observes the CT's lessons, interacts with the CT's students individually and in small groups, and teaches the CT's in whole class format.

[9] TODOS-Mathematics for All is a non-profit organization whose mission is to advocate for an equitable and high quality mathematics education for all students, in particular Latino/Hispanic students, by increasing the equity awareness of educators and their ability to foster students' proficiency in rigorous and coherent mathematics. For more information and to join, go to: http://www.todos-math.org/mc/page.do.

[10] The California Beginning Teacher Support and Assessment (BTSA) Induction Program provides formative assessment, individualized support and advanced content for newly-credentialed, beginning teachers, and is the preferred pathway to a California Clear Teaching Credential. For more, go to http://www.btsa.ca.gov/BTSA_basics.html .

A CSTP-Based Portfolio for a Secondary Mathematics Methods Course

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One function of a secondary mathematics methods course is to take pre-service teachers who view themselves as mathematics *students* and help them view themselves as mathematics *professionals*. How can methods instructors prepare pre-service teachers to be professionals? This paper will describe a culminating assignment for a secondary mathematics methods course with that goal in mind. The assignment is the creation of a professional portfolio that demonstrates pre-service teachers' progress towards fulfilling the California Standards for the Teaching Profession (CSTPs). The portfolio allows pre-service teachers to collect and display evidence of their professional mathematical practice.

While a portfolio is not an unusual assessment method for a methods course, this assignment directly addresses the needs of California teachers. The CSTPs are the standards that the California Beginning Teacher Support and Assessment Induction Program (BTSA) requires professionals to address in their first two years of teaching in order to obtain a clear credential. BTSA provides assessment, support, and content for newly-credentialed teachers and is a common pathway to a California Professional (Clear) Teaching Credential. As a result, many school districts evaluate their teachers using the CSTPs. The portfolio includes the added benefit that students in a credential program authorized by the California Commission on Teacher Credentialing are required to demonstrate their knowledge of the CSTPs before leaving the program; this mathematics portfolio serves as a foundation for the portfolio students will use at the end of their credential year.

This assignment, by using the CSTPs, focuses on the needs of California teachers, but the assignment could be modified to suit the needs of teachers in other states as well. A portfolio could be organized around any set of standards. Those outside of California might choose to organize such an assignment around the six NCTM principles explained in *Principles and Standards for School Mathematics* (2000) or around the NCTM *Professional Standards for Teaching Mathematics* (1991).

The six CSTP standards (California Commission on Teacher Credentialing, 1997) are:

1. Standard One: Engaging and supporting all students in learning;

2. Standard Two: Creating and maintaining effective environments for student learning;
3. Standard Three: Understanding and organizing subject matter for student learning;
4. Standard Four: Planning instruction and designing learning experiences for all students;
5. Standard Five: Assessing student learning;
6. Standard Six: Developing as a professional educator.

This portfolio assignment is used at an institution that offers a program in undergraduate mathematics education, generally taken in four years, followed by a secondary credential, taken in a post-baccalaureate credential year. In the credential year the mathematics pre-service teachers are among a large cohort of 40–70 pre-service teachers from varied disciplines, and the mathematics methods course is the only course that is exclusively for the small cohort (usually 6–12) of pre-service secondary mathematics teachers. The methods course consists of two one-hour units of a 17.5 unit fall curriculum. The pre-service teachers are engaged in the observation and participation phase of student teaching; that is, they are observing and assisting six hours per week in the classroom where they will complete full-time student teaching in the spring.

The Portfolio

Throughout the semester, the pre-service teachers collect their written work. The work assigned in the methods course includes lesson plans, essays, professional development work, and other professional writings. All of the assignments are designed to address the goal of guiding these pre-service teachers in the transition from student to professional. The assignments include

1. Sample lesson plans;
2. Reflective essays;
3. A synopsis of a professional development conference;
4. The creation of a professional development activity;
5. Assessment items; and
6. Other professional documents, like teaching statements and classroom management plans.

The lesson plans demonstrate the pre-service teachers' implementation of content material in the curriculum. The plans allow the pre-service teachers to demonstrate specific teaching strategies and tools, such as lessons that include adjustments for English Language Learners, lessons that demonstrate the appropriate use of technology, and lessons in which students use manipulatives.

The reflective essays are designed to give pre-service teachers the opportunity to read and reflect on the professional documents that influence the work of mathematics teachers in California. The essays introduce the pre-service teachers to the professional documents *Principles and Standards for School Mathematics* (NCTM, 2000) and *Mathematics Framework for California Public Schools* (California Department of Education, 2005). In another essay, designed to include a national perspective, the pre-service teachers analyze an aspect of *Knowing and Teaching Elementary Mathematics* (Ma, 1999).

The pre-service teachers also engage in formal participation in mathematics professional development. They attend an NCTM affiliate's annual meeting, a one-day local event, and write a summary of the sessions they attend. This conference requires them to interact with other mathematics teaching professionals. They attend workshops and lectures designed for practicing mathematics teachers. A primary goal of this assignment is for them to witness mathematics teaching professionals engaging in life-long learning.

In another assignment they create an activity that could be used at a mathematics professional development workshop. They first read the essay *Mathematics for Teaching* (Cuoco, 2003), emphasizing to them the need for a strong connection between undergraduate mathematics and school mathematics. Then they choose a topic from undergraduate mathematics, analyze its connection to school mathematics, and create an activity that would allow them to teach a lesson to secondary schoolers based on what they know about this connection (Burroughs, 2007).

They consider issues of assessment in mathematics. In one assignment they design a grading rubric for a mathematics project. In another they rewrite textbook publishers' test bank questions as open-ended assessment items.

Finally, the pre-service teachers include in their portfolios other work of a mathematical or professional nature. They write teaching statements in preparation for completing job applications. Each writes a sample letter that outlines his or her classroom management plan, with the intention that this letter will be sent home with students when the pre-service teacher takes on full-time student teaching in the spring semester.

In the final methods class meeting, the pre-service teachers reconsider all assignments and activities they have engaged in during the semester. In small groups, they think critically about how these assignments show their progress towards mastery of the six CSTPs. This in-class assignment serves as the framework for the organization of their portfolios. The pre-service teachers have the responsibility of determining how the course's assignments help them develop their professionalism, and this in itself serves to increase their professionalism. They do not consider the assignments to be something they are doing because the instructor told them to, but rather, they have done these assignments so that they can demonstrate their competency as professionals.

The pre-service teachers organize their portfolios with an introduction, a table of contents, a brief reflection about how each piece demonstrates one of the CSTPs, and a conclusion that highlights their analysis of their own strengths as teachers and areas where they require further growth.

The Portfolio Assessment

The instructor assesses the portfolio using the rubric in Figure 1. (The article *Everyday Rubric Grading* (Stutzman & Race, 2004) explains the EMR grading scale). Portfolios determined to be "excellent" according to the qualifications listed in the rubric are scored "E," which is equivalent to an A. Portfolios determined to have "met expectations" are scored "M," which is equivalent to a B. Portfolios which do not meet expectations are scored an "R," and the student must redo the portions of the portfolio that are below expectations. Portfolios successfully redone will be assigned a score of "M." In the five years the author has used this portfolio assignment, all portfolios submitted have earned either an "E" or an "M."

The EMR grading scale works well for the portfolio and accompanying interview because of the nature of the portfolio assignment. The goal is for the pre-service teachers to view themselves as professionals. Scoring the portfolios as "excellent" or "met expectations" gives pre-service teachers a realistic sense of how their professional work will be viewed. Also, the EMR scale is appropriate because each of the assignments has already been scored individually, and so a detailed scoring of each assignment is not necessary.

How individual pre-service teachers choose to categorize assignments is not of primary importance. That is, a pre-service teacher could include a lesson plan as evidence of CSTP Standard Four: Planning instruction and designing learning experiences for all students, or the same lesson plan as evidence of CSTP Standard One: Engaging and supporting all students in learning. The instructor evaluates the professional quality of the item and the appropriateness of the pre-service teacher's reflection about that assignment. The value in the portfolio for these incipient mathematics teachers is their reflection on their professional work. The minimum requirement is that the portfolio address four of the standards, and those portfolios that address all six are eligible for As. Though it is not required, pre-service teachers can choose to include evidence from assignments in other education or mathematics courses.

As a final exam in the course, each pre-service teacher attends a one-on-one interview with the instructor. Here, they answer questions similar to those in Figure 2. They are encouraged to use their portfolios and the evidence contained therein to support their answers to the interview questions, much as they would do in a professional job interview.

Conclusion

This portfolio serves one of the main goals in the secondary mathematics methods course: that is, to have pre-service teachers view themselves and their work from the perspective of professional educators. This assignment builds a foundation for their future work as professionals. The pre-service teachers use these portfolios to fulfill a requirement for the credential program, in job interviews, and to obtain a clear credential. The assignment serves the needs of these teaching professionals.

Portfolio Rubric

E The portfolio is focused and well-prepared. All necessary portions are included. Reflections show a clear understanding of NCTM and CA Standards and a reasonable interpretation of balance in mathematics instruction (concepts, skills, problem-solving) and the corresponding roles of teachers and students in this balance. The portfolio addresses all CSTP categories thoughtfully.

M The portfolio is well-prepared. All necessary portions are included. Reflections show a working understanding of NCTM and CA Standards and a reasonable interpretation of balance in mathematics instruction (concepts, skills, problem-solving) and the corresponding roles of teachers and students in this balance. The portfolio addresses at least four CSTP categories.

R The portfolio lacks direction. There is little evidence of knowledge of the content expectations in the NCTM and CA Standards and a reasonable interpretation of balance in mathematics instruction (concepts, skills, problem-solving) and the corresponding roles of teachers and students in this balance. Fewer than four CSTP categories are addressed.

Interview Questions

1. What is important mathematics for school students to study? What should students understand, be able to do, and be able to apply? Focus on students in the classes where you are placed.
2. How do you (will you) support students in learning this mathematics? What strategies do you use that are specific to learning mathematics? Be sure to consider conceptual development, algorithmic skills, and problem-solving.
3. What aspects of your teaching abilities are you most confident about/excited about/proud of? What is hardest for you?

Interview Rubric

E The response is focused and well prepared. The response reflects a good working knowledge of the content expectations in the NCTM and CA Standards and a reasonable interpretation of balance in mathematics instruction (concepts, skills, problem-solving) and the corresponding roles of teachers and students in this balance.

M The response is well prepared, reflecting some knowledge of the content expectations in the NCTM and CA Standards and a reasonable interpretation of balance in mathematics instruction (concepts, skills, problem-solving) and the corresponding roles of teachers and students in this balance.

R The response lacks direction. There is little evidence of knowledge of the content expectations in the NCTM and CA Standards and a reasonable interpretation of balance in mathematics instruction (concepts, skills, problem-solving) and the corresponding roles of teachers and students in this balance.

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